Fractional Order Model for the Thermal Behavior of Bipolar Transistors

H. Mnif, T. Zimmer, J.L. Battaglia

University of Bordeaux 1, France

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Outline

- Introduction, self-heating
- Measurement set-up
- The new self-heating model
- Network representation
- Results, conclusion

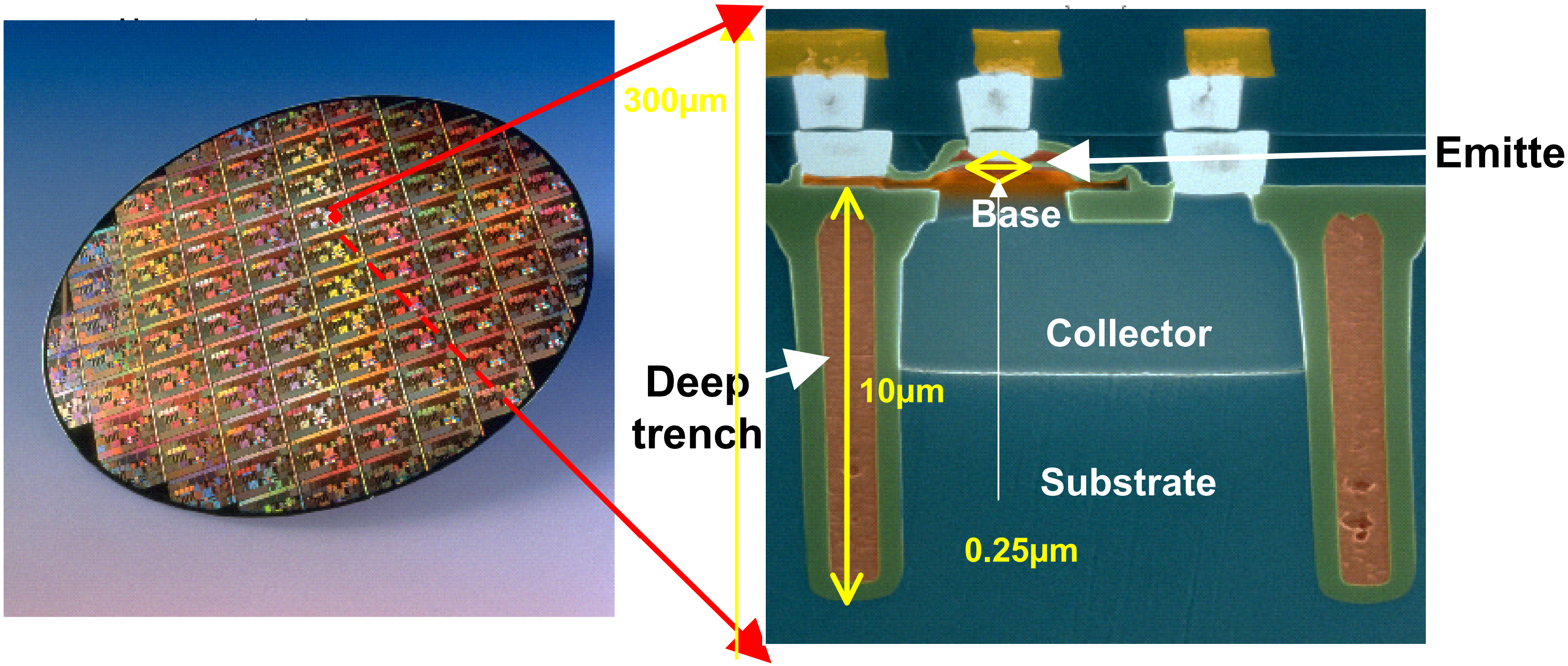
Introduction

- Self-heating:
  - Heating of the device due to its power dissipation
- Bipolar transistor:
  - First order approximation:

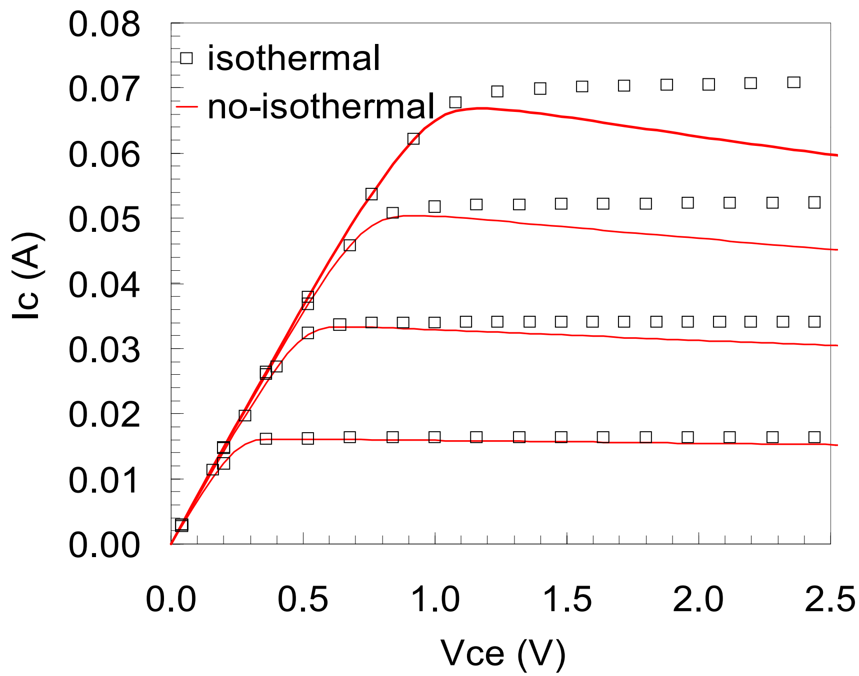
    $P = I_C V_{CE} + I_B V_{BE}$
  - Effect accentuated in recent SiGe HBT technology: deep trench isolation

Introduction

200mm Wafer   SiGe SEM Cross-Section



Introduction



- Self-heating affects electrical characteristics
- Thermal behavior of integrated circuits can be effectively improved by means of layout optimization and technology

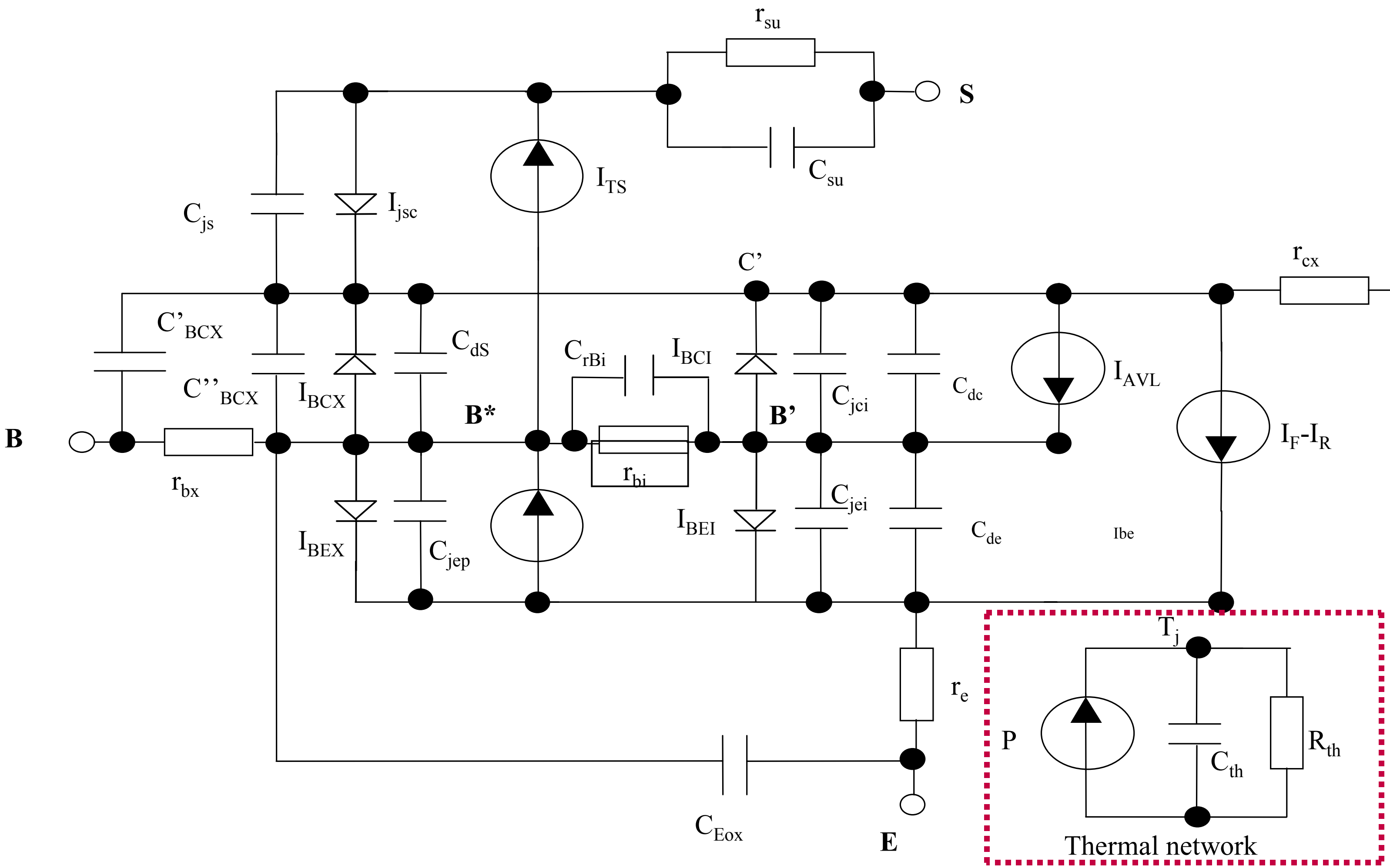
→ **Consistent electro-thermal model**

Electro-thermal simulation approaches

- Physical device simulation including self-heating
- Electro-thermal simulation using circuit simulation for the electrical analysis. Thermal analysis can be carried out by means of
  - Numerical method
  - Analytical method
  - Compact thermal models

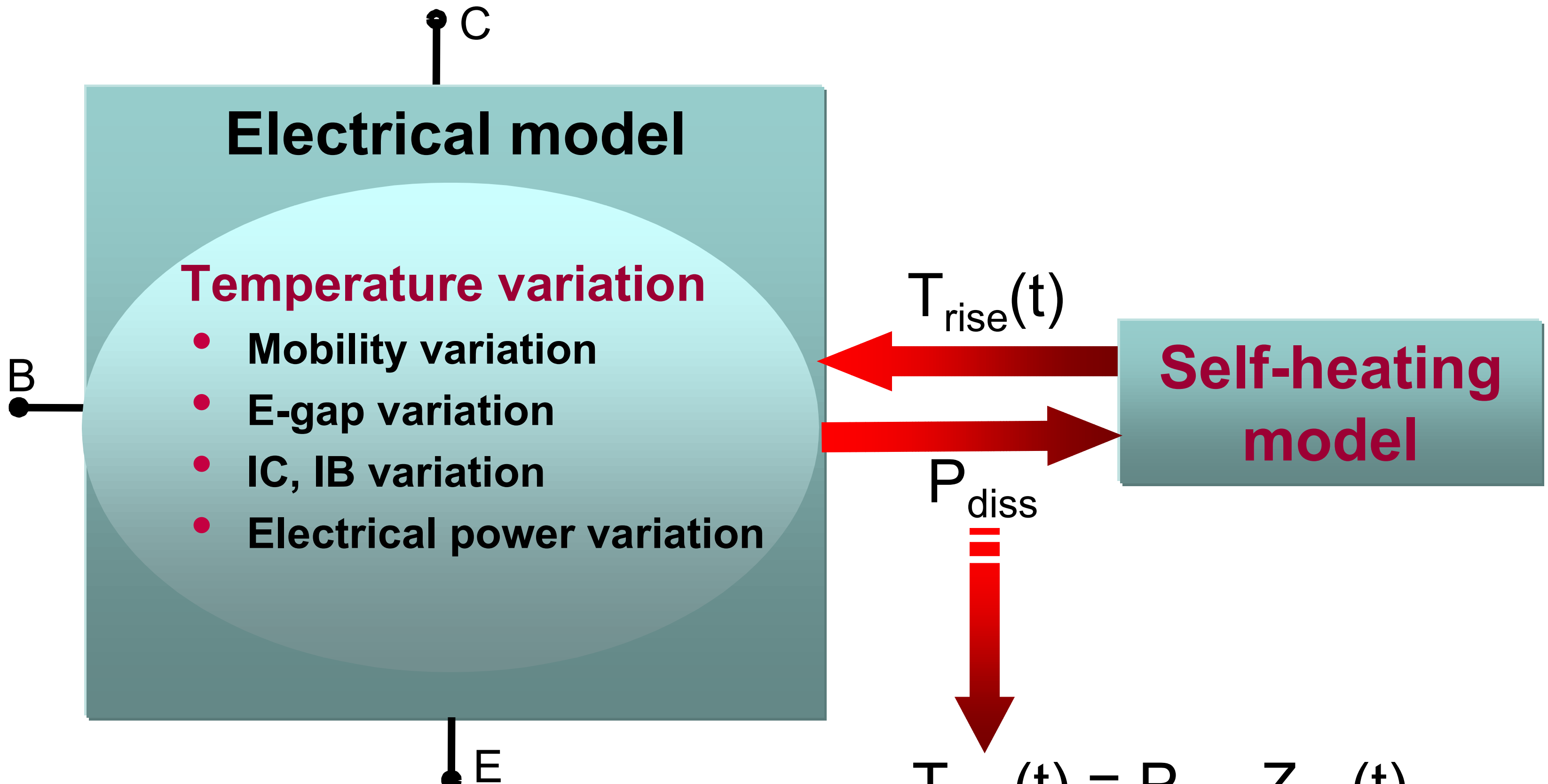
Temperature at device nodes and heat fluxes are expressed by simple relations that can be represented by simple equivalent electrical networks

Compact HICUM model



Introduction: coupling

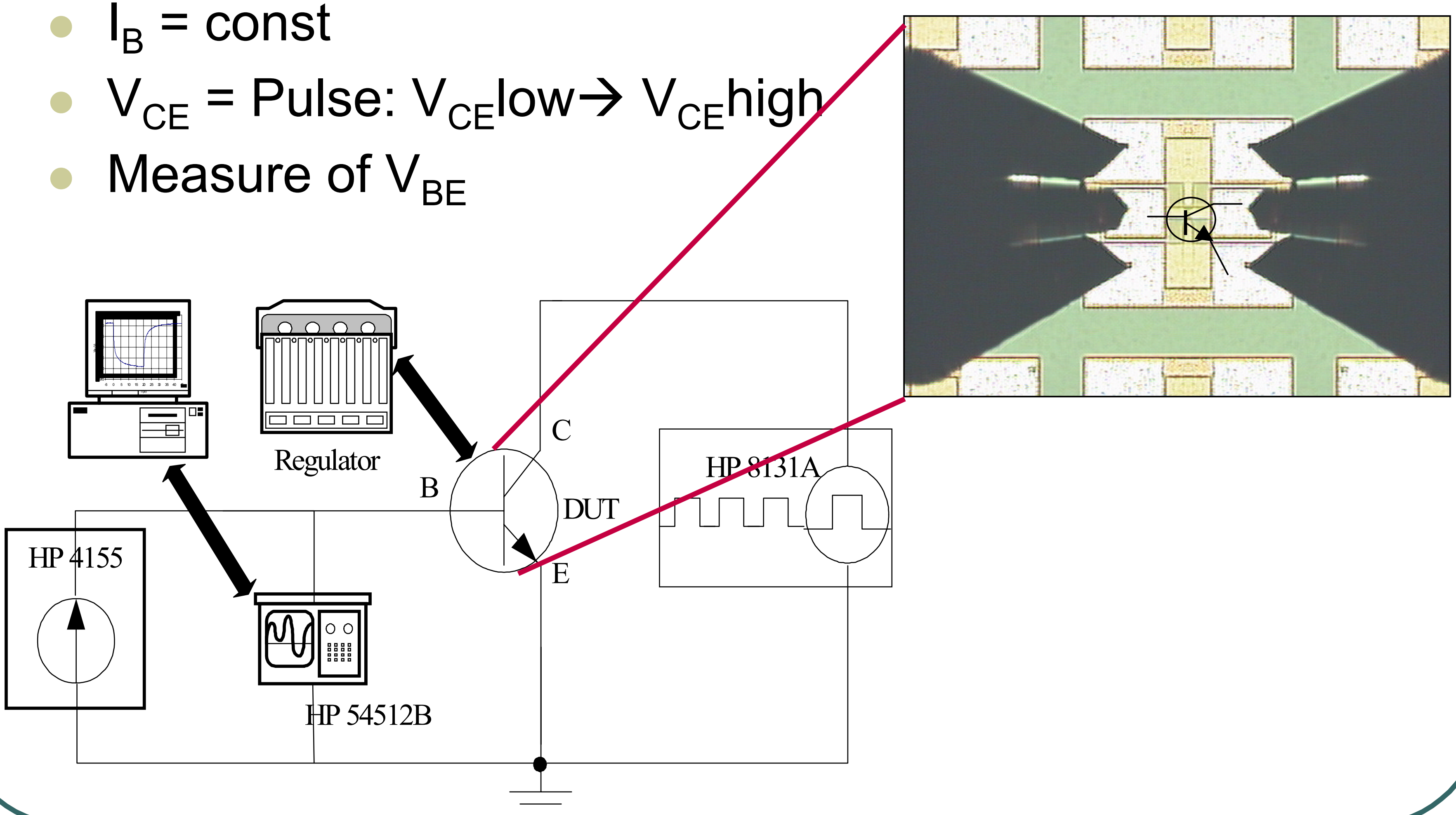
- Electro-thermal coupling



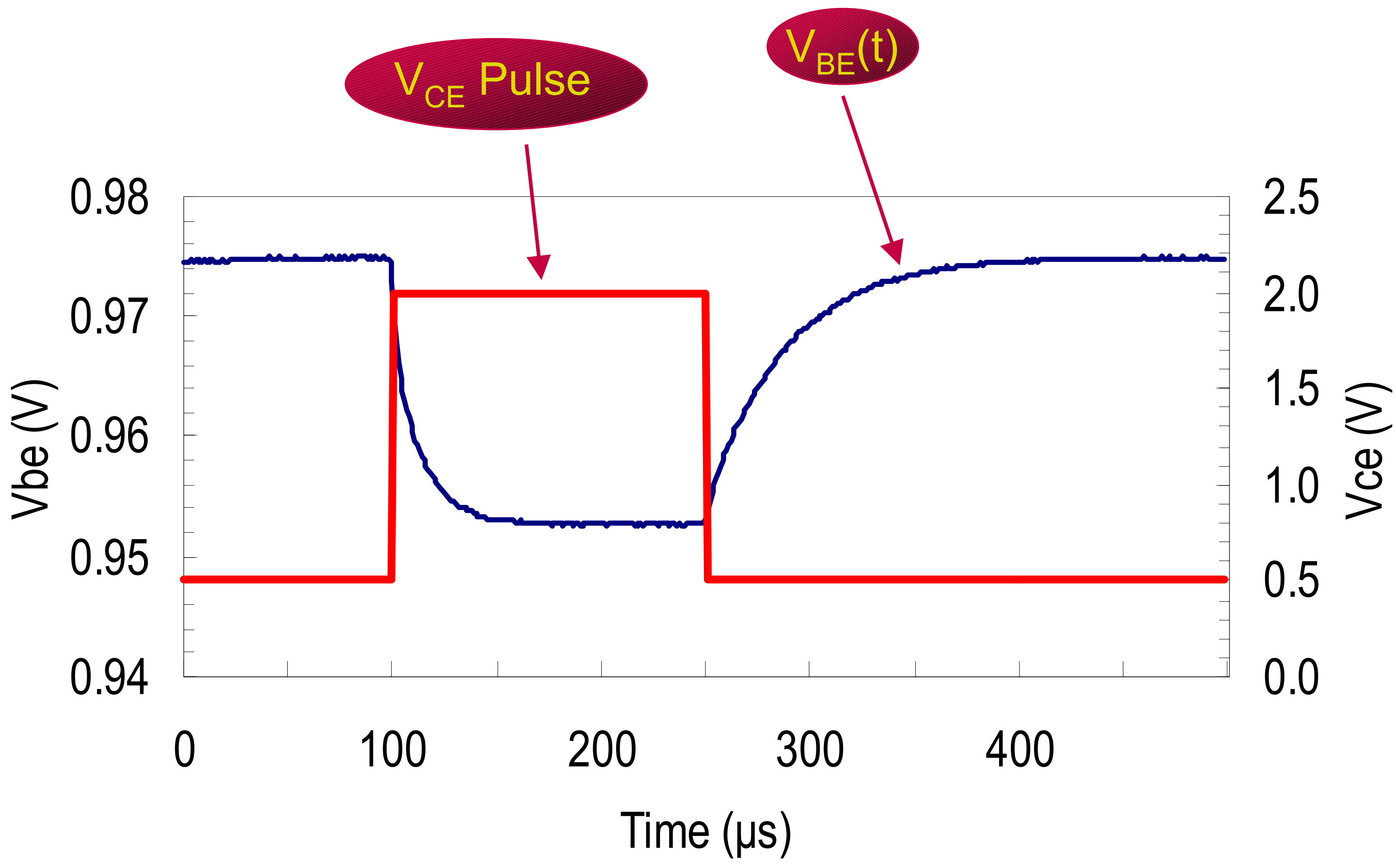
$$T_{rise}(t) = P_{diss} \, Z_{TH}(t)$$

Measurement set-up: step 1

- I_B = const
- V_{CE} = Pulse: V_{CE}low→ V_{CE}high
- Measure of V_{BE}

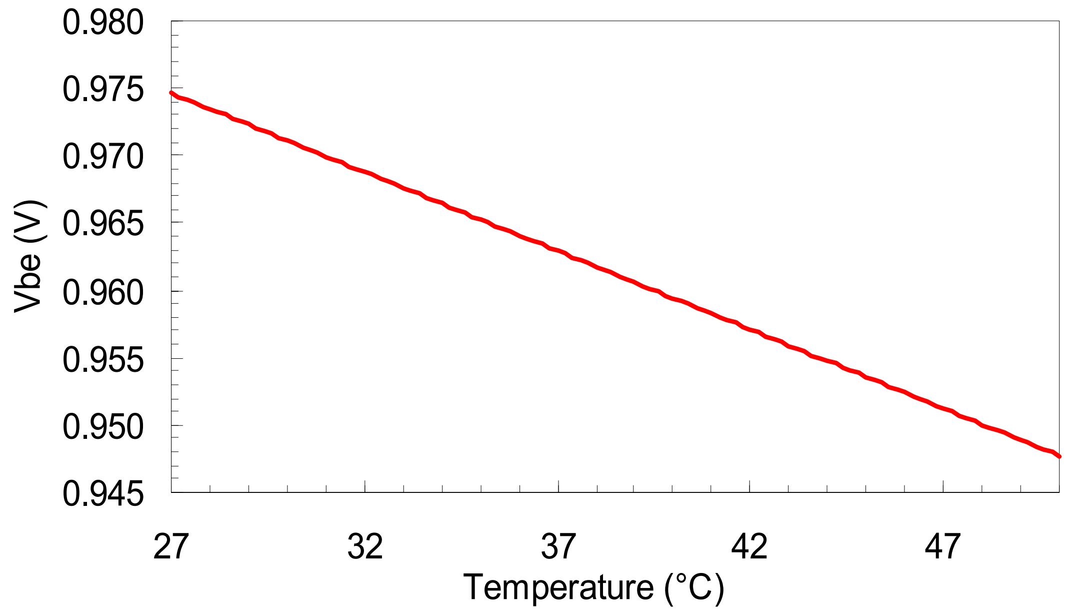


Measurement set-up: step 1



Calibration: Measure V_{BE}(T), step 2

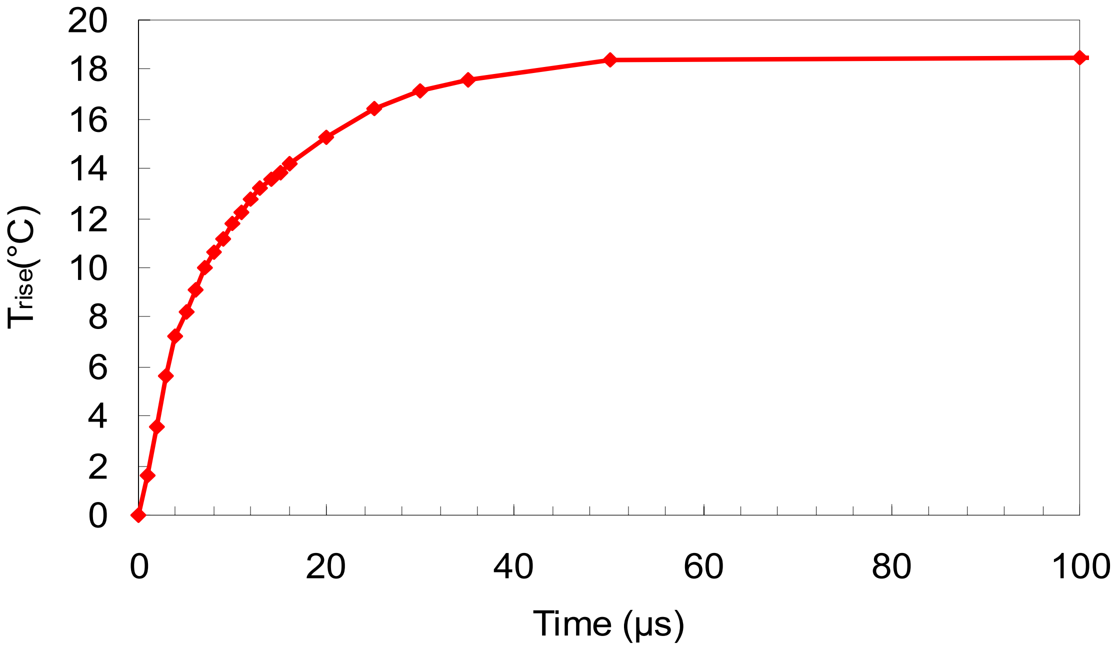
- I_B = const
- $V_{CE} = V_{CE}low$
- Variation of T
  - 27°C –> 50°C
- Measure of V_{BE}



Dynamic behaviour: T_{rise}(time), step 3

Calibration

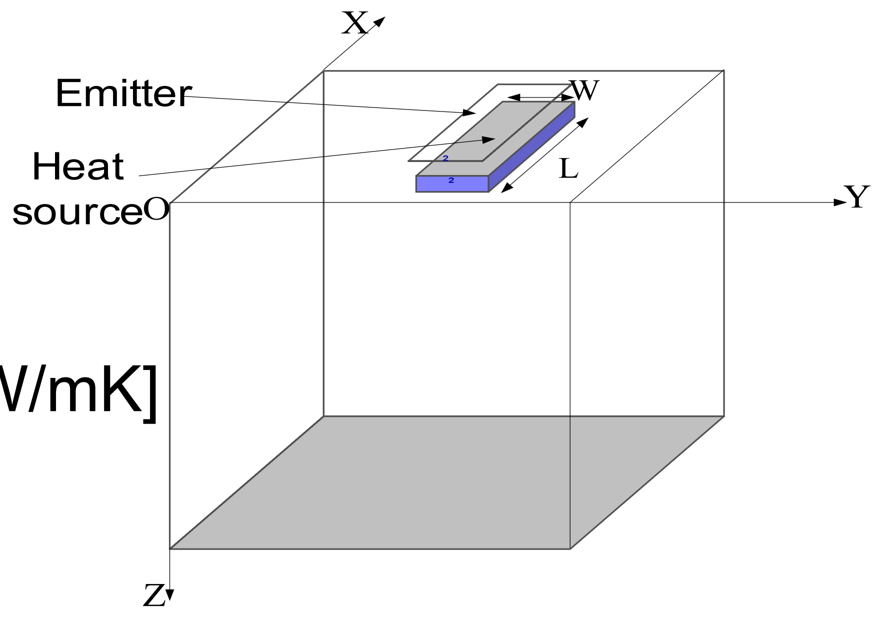
- From $V_{BE}(t)$
- And $V_{BE}(T)$
- –> T_{rise} (t)



HBT SiGe: Thermal problem

- Heat transfer in semi-conductor occurs mainly through the conduction mode
  - Heat equation

$$c\rho\Delta T = \nabla\left(\lambda(T)\nabla T\right)$$



  - T = temperature [°C or K]
  - λ = thermal conductivity [W/mK]
  - C = specific heat [J/KgK]
  - ρ = density [Kg/cm^3]

Problem complexity

- 3-dimensional heat spreading

→ 3-dimensional problem

- Thermal conductivity is temperature dependant

→ Non linear problem

➡ Approximations and simplifications

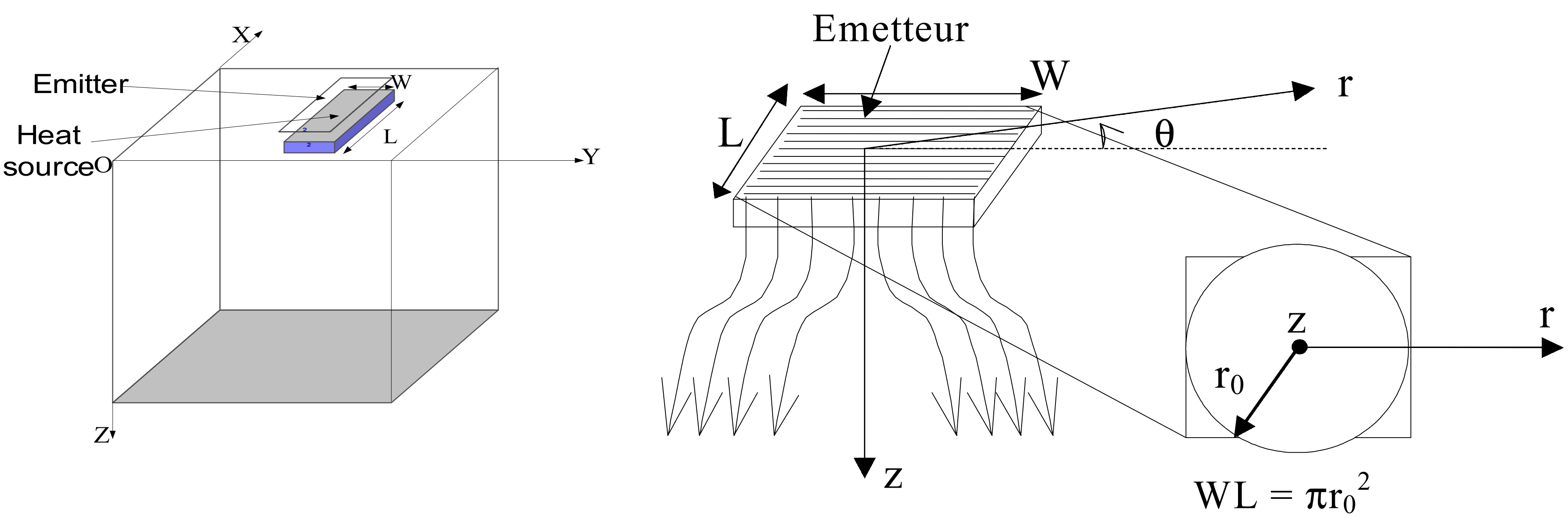
Axisymmetrical 2-dimensional Structure

- Neglected surfacing temperature gradient

  → Only vertical flux propagation is considered
- No θ-dependence

  → Symmetrical structure
- No temperature dependence of the thermal conductivity

Structure in cylindrical co-ordinates



$$\frac{\partial^2 T_{rise}(t,r,z)}{\partial r^2} + \frac{1}{r}\frac{\partial T_{rise}(t,r,z)}{\partial r} + \frac{\partial^2 T_{rise}(t,r,z)}{\partial z^2} = \frac{1}{a}\frac{\partial T_{rise}(t,r,z)}{\partial t}$$

a = $\lambda/c\rho$ = thermal diffusivity [m^2/s]

Boundary conditions (1)

- The symmetry involves directly that the temperature in the r-θ plane is maximal at r=0,

$$\frac{\partial T}{\partial r} = 0 \;\; for \;\; , r = 0$$

- For the special case of an ideal heat sink, we impose Dirichlet boundary condition,

$$T = T_{\infty},\ for\ r \to \infty,\ z \to \infty$$

- A variable heat flow was applied within the active region at the top surface,

$$-\lambda \frac{\partial T}{\partial z} = q(r,t)\ for\ z = 0,\ 0 < r < r_0$$

Boundary conditions (2)

- No heat flow is assumed outside the active region at the top surface,

$$-\lambda \frac{\partial T}{\partial z} = 0 \ \ for \ z = 0, r > r_0$$

- Initial condition

$$for \quad t = 0, \quad T = T_\infty$$

Analytical problem resolution

- Calculation of the thermal impedance
- $T_{rise}(t) = P_{diss}\, Z_{TH}(t)$
- Transform into the Laplace domain and solution of differential equation:
- $$Z_{TH}(p) = \frac{R_{TH}}{\left(1 + \sqrt{R_{TH} C_{TH}}\sqrt{p}\right)}$$

← **½ order behavior**

$$\begin{cases} R_{TH} = \dfrac{8}{3\pi^2 \lambda r_0} = \dfrac{8}{3\pi^2 \lambda}\sqrt{\dfrac{\pi}{WL}} \\ C_{TH} = \dfrac{8}{3}\rho c r_0^3 \end{cases}$$

Transient response

- The device temperature rise transient response is the inverse Laplace transform of $Z_{TH}(p)$:

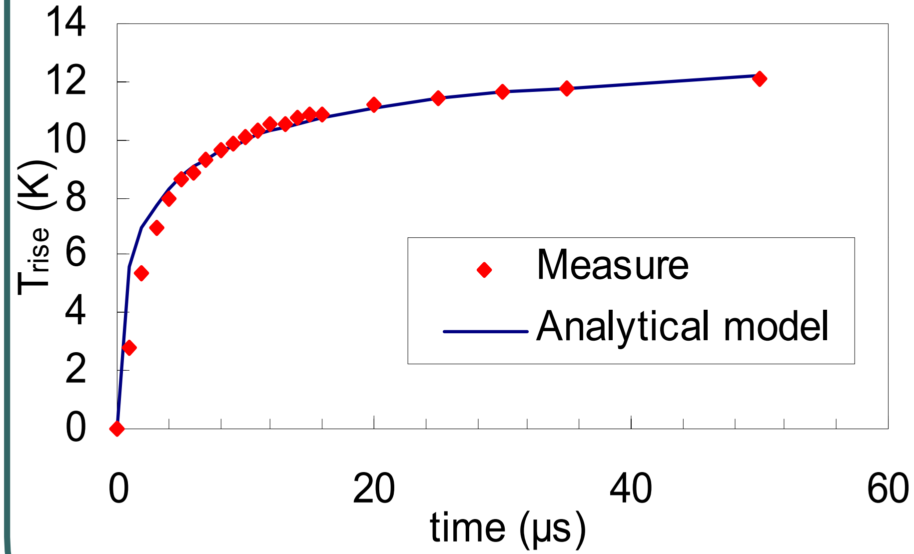
$$T_{rise}(t) = L^{-1}\{Z_{TH}(p)P_{diss}\}$$

$$T_{rise}(t) = R_{TH}P_{diss}\left(1 - exp\left(\frac{t}{R_{TH}C_{TH}}\right)erfc\left(\sqrt{\frac{t}{R_{TH}C_{TH}}}\right)\right)$$

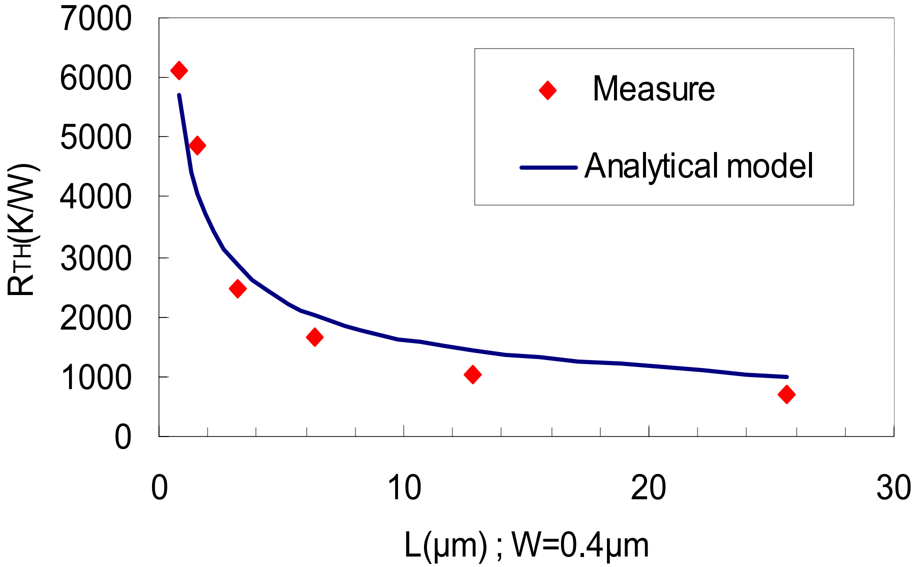
Model validation

- SiGe BiCMOS technology from STMicroelectronics ($\beta = 300$ and $f_T = 160GHz$)

$T_{rise}(t)$



R_{TH}



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Electrical analogy

- Model inside a circuit design environment

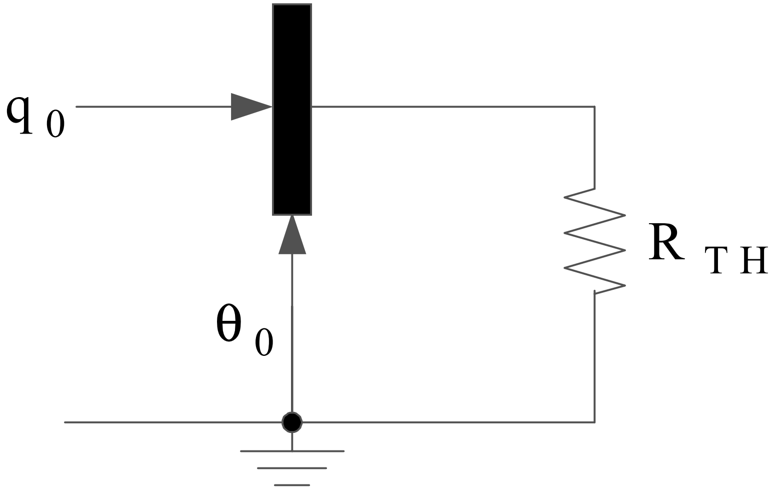
→ Network representation for SPICE compatibility

Thermal conductivity	Electrical conductivity
Temperature T (K)	Voltage V (V)
Thermal conductivity λ (W/mK)	Electrical Conductibility σ $(\Omega m)^{-1}$
Heat flux P (W)	Intensity I (A)
Thermal capacity C_{TH} (J/K)	Electrical capacity C (F)
Thermal resistance R_{TH} (K/W)	Electrical resistance R (Ω)

Network representation

- Thermal impedance: $$Z_{TH}(p) = \frac{R_{TH}}{\left(1+\sqrt{R_{TH}C_{TH}}\sqrt{p}\right)}$$
- In the steady-state ($t \to \infty$), that is for $p \to 0$

$$Z_{TH}(p) \to R_{TH}$$



Network representation

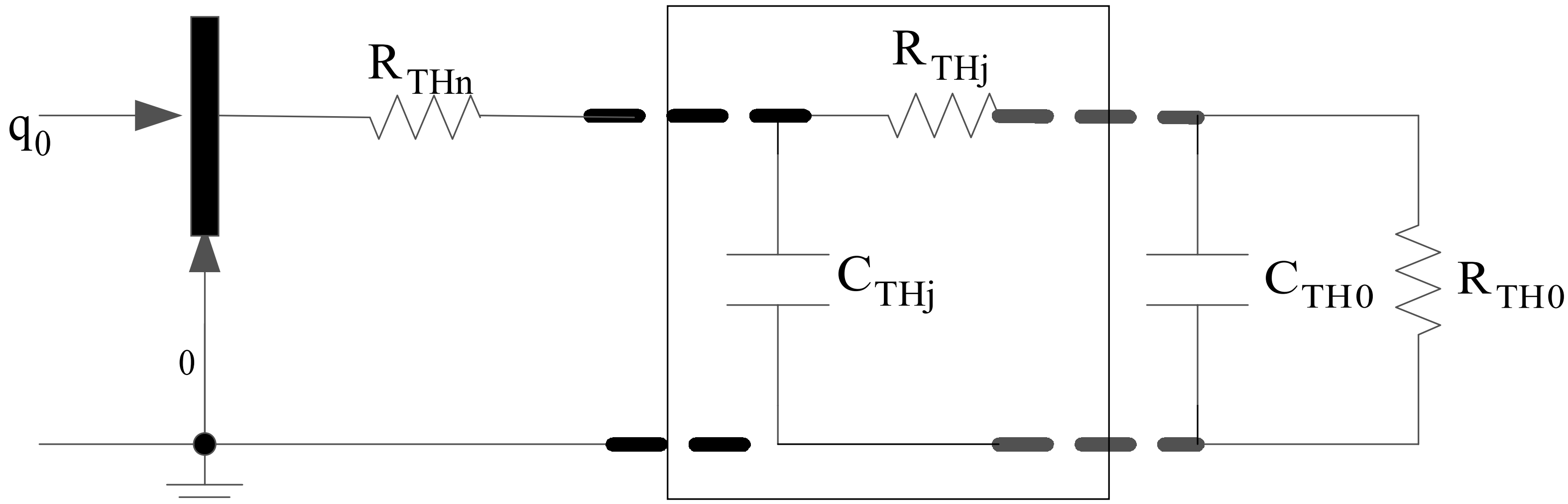
- In the transient-state ($t \to 0$), that is for $p \to \infty$

$$Z_{TH}(p) \to \sqrt{\frac{R_{TH}}{C_{TH}}}\sqrt{\frac{1}{p}}$$

- Find an equivalent network for representing the thermal impedance both, for the steady and the transient states

Linear network

- Linear network



$$R_{TH0} = R_{TH1} = R_{TH2} = ...= R_{THn-1} = \frac{R_{TH}}{n}$$

$$R_{THn} = \frac{1}{2n} R_{TH}$$

$$C_{TH0} = C_{TH1} = C_{TH2} = ...= C_{THn-1} = \frac{C_{TH}}{n}$$

Linear network

$$Z_{TH}(f) = R_{TH} \sqrt{\frac{1}{j\frac{f}{f_c}}} \sqrt{1 + \frac{j\frac{f}{f_c}}{4n^2}} \left(2 \Bigg/ \left(1 + \left[\left(\sqrt{\frac{4n^2}{j\frac{f}{f_c}} + 1} - 1 \right) \Bigg/ \left(\sqrt{\frac{4n^2}{j\frac{f}{f_c}} + 1} + 1 \right) \right]^{2n+1} \right) - 1 \right)$$

$$f_c = \frac{1}{2\pi R_{TH} C_{TH}}$$ cut-off frequency

Linear network

- First case: $f \ll f_c$,

$$\rightarrow \; Z_{TH} \approx R_{TH}$$

- Corresponds to the thermal impedance in the steady state, network can be used for the steady state

Linear network

- Second case: $f \gg f_c$ and $f/f_c \ll 4n^2$

$$\rightarrow \quad |Z_{TH}| \approx \sqrt{\frac{R_{TH}}{C_{TH}}} \frac{1}{\sqrt{f}}$$

- Corresponds to the thermal impedance in the transient state. Network represents a fractional order model for this frequency band and under the assumption that n is rather big

Linear network

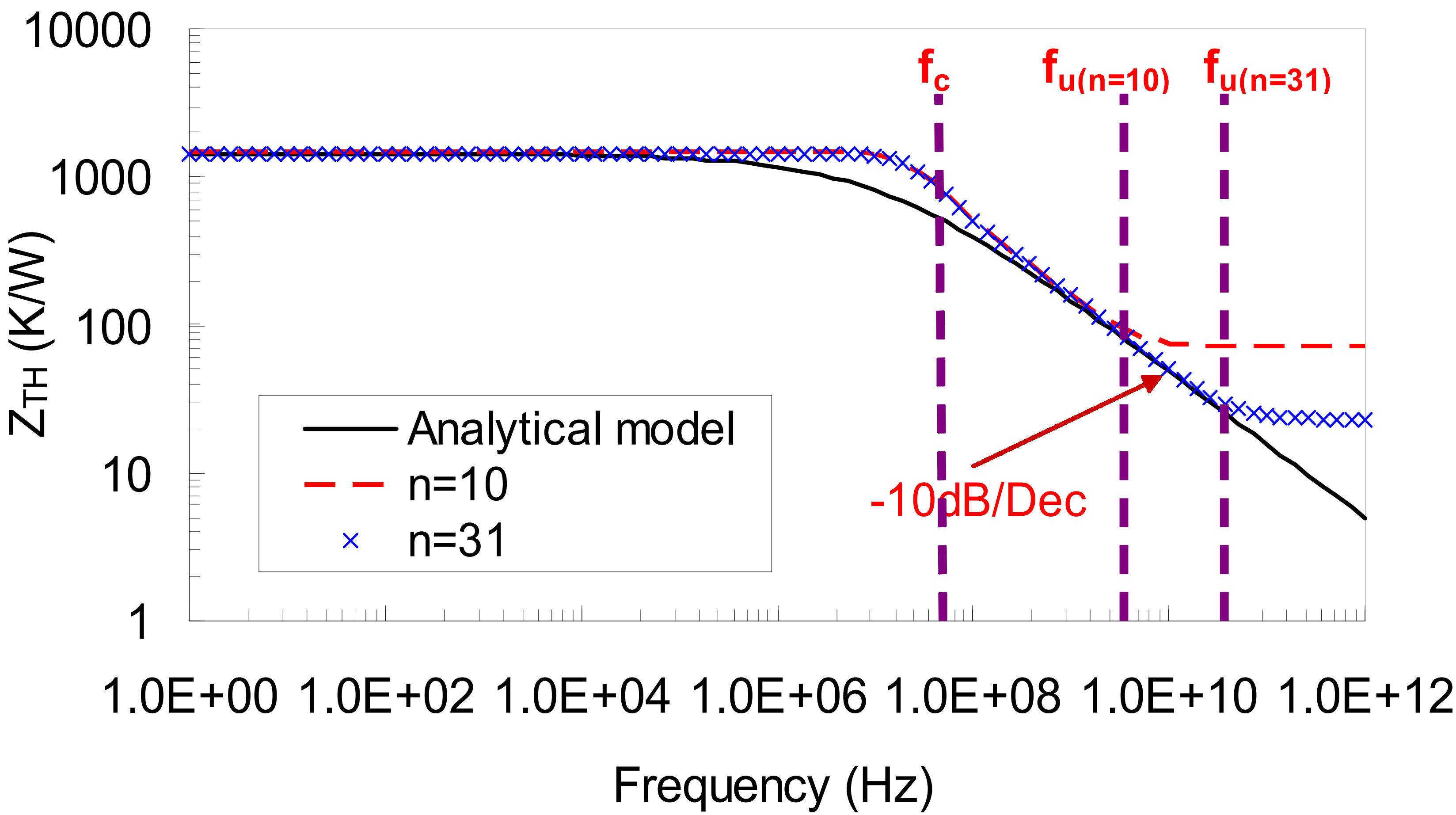
- Third case: $f >> f_c$ and $f/f_c >> 4n^2$,

$$\rightarrow \quad |Z_{TH}| \cong \frac{1}{4n^2} R_{TH}$$

- Upper frequency limit f_u of network for representing the thermal impedance:

$$f_u = 4n^2 f_c$$

Linear network



Linear network

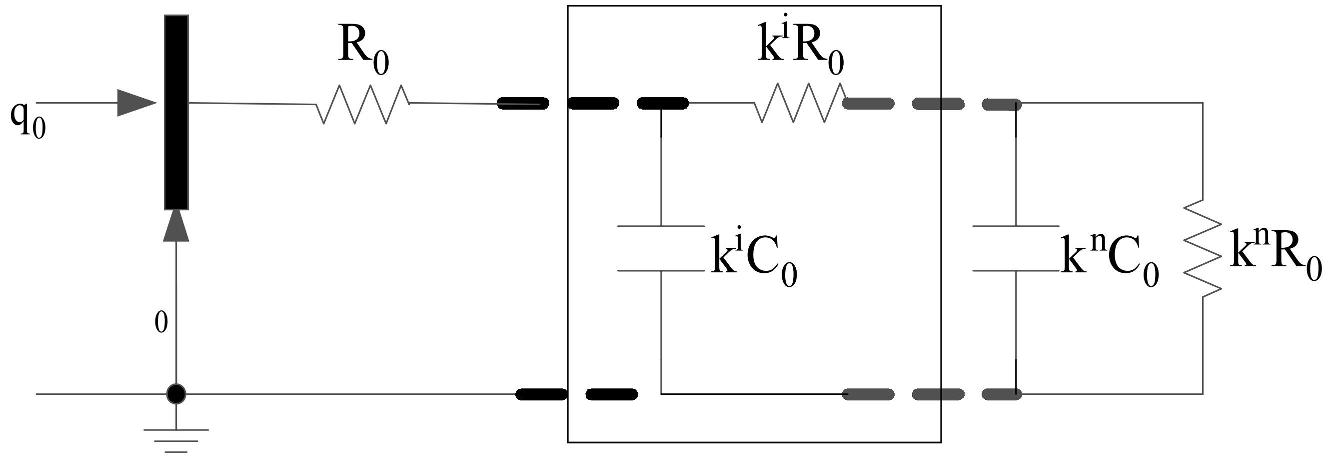
- $f_u = 4n^2 f_c \quad \rightarrow \quad \log \frac{f_u}{f_c} = 2 \log(2n)$
- Increasing the frequency fit by two decades

$\rightarrow$ Increasing n by 1decade

$\rightarrow$ For an accurate representation over a large frequency range, a great number of cells is necessary

$\rightarrow$ Drawback for usability in practical applications due to complexity and CPU simulation time

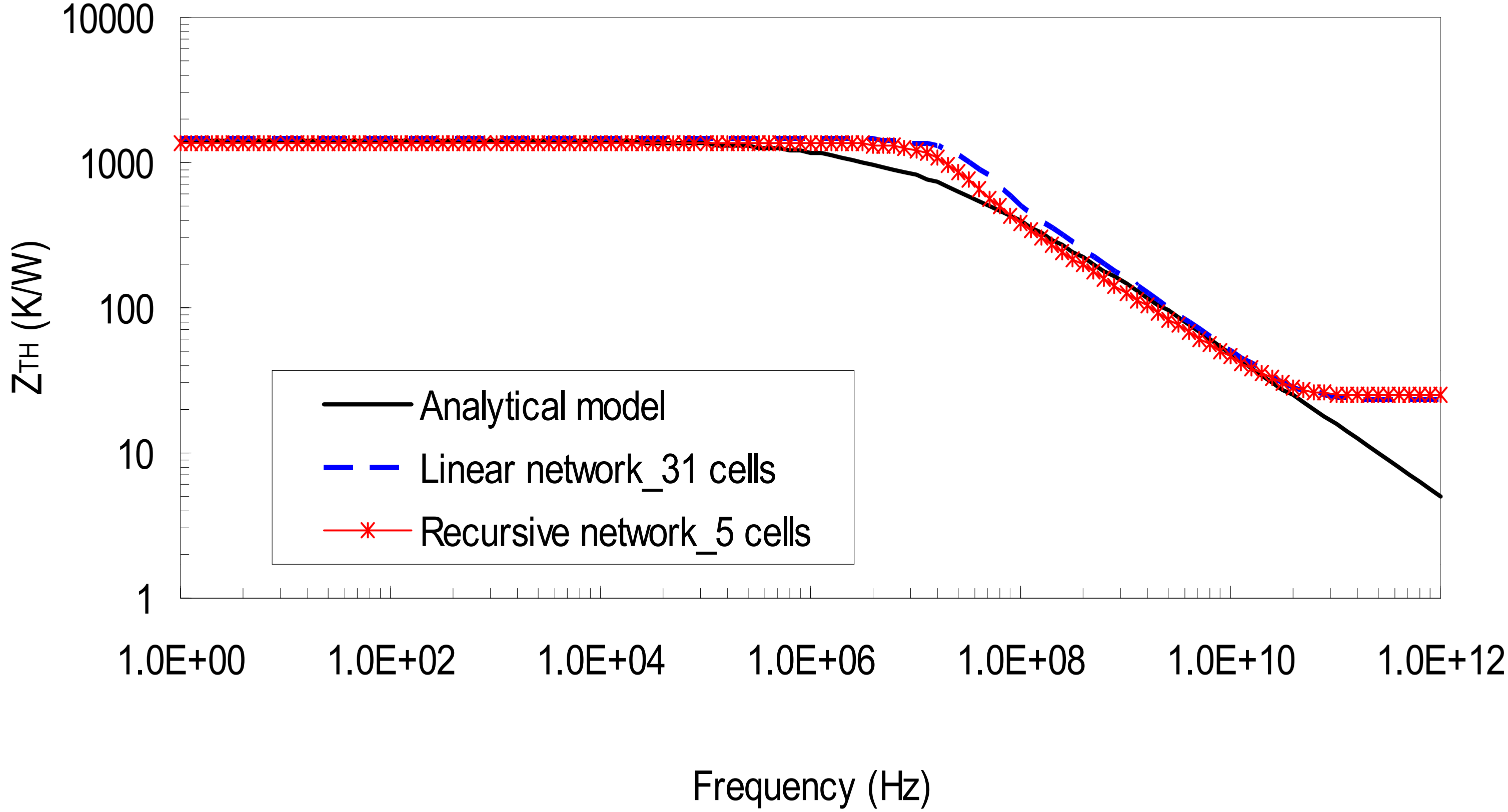
Recursive network



- k: recursivity coefficient, an additional degree liberty

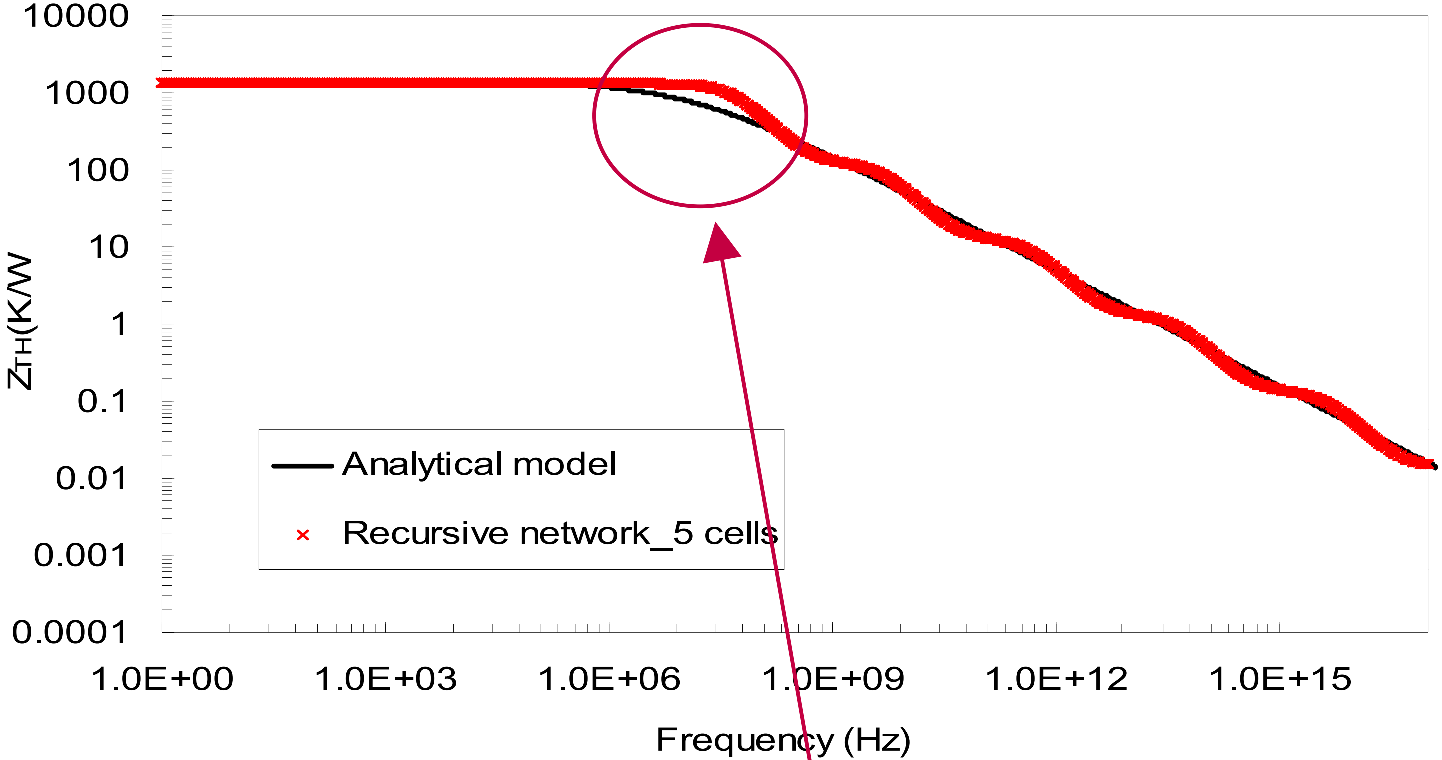
→ $$f_u \approx k^{2n} f_c$$

Comparison



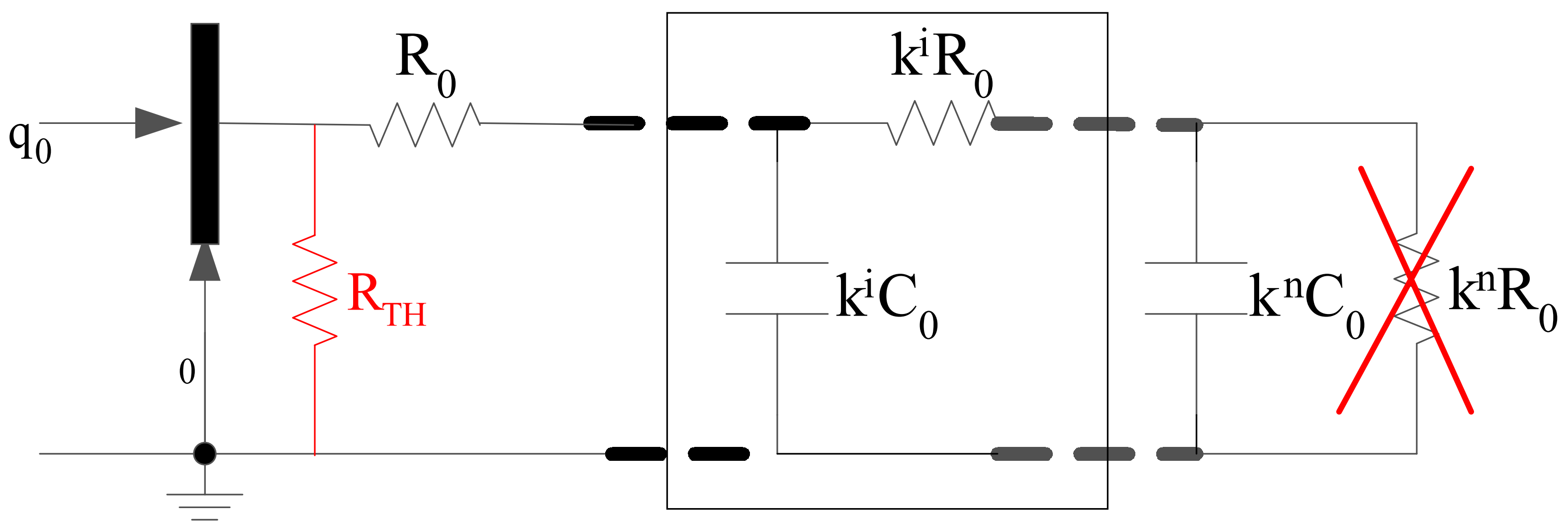
- $f_{u(n=31)}$(linear)=$f_{u(n=5)}$(recursive)

Comparison



Imprecision near of cut-off frequency and oscillation

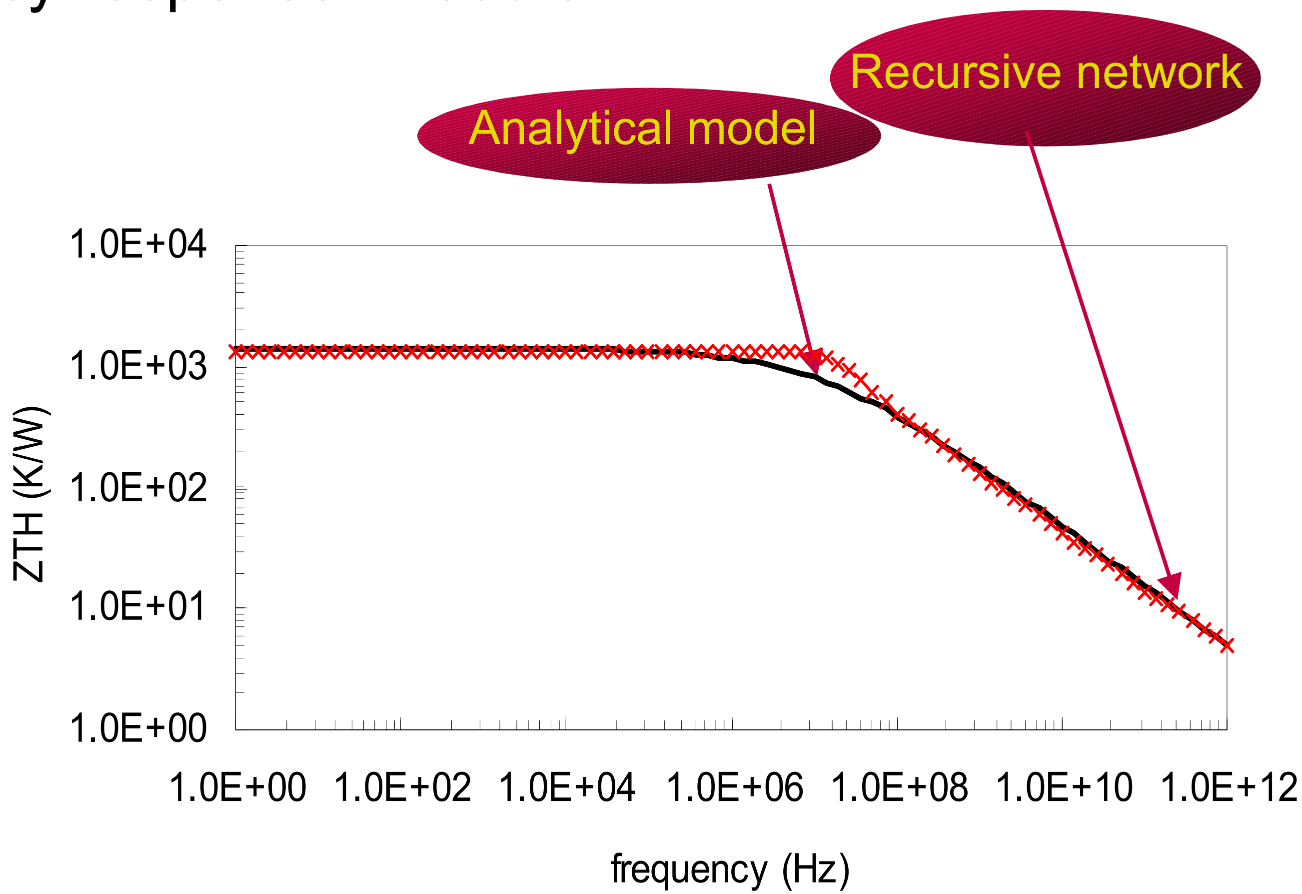
Improved recursive network



Comparison

- Frequency response: Module

- λ= 148 W/mK
- C=710 J/kgK
- ρ = 2330 kg/m^3
- n = 10



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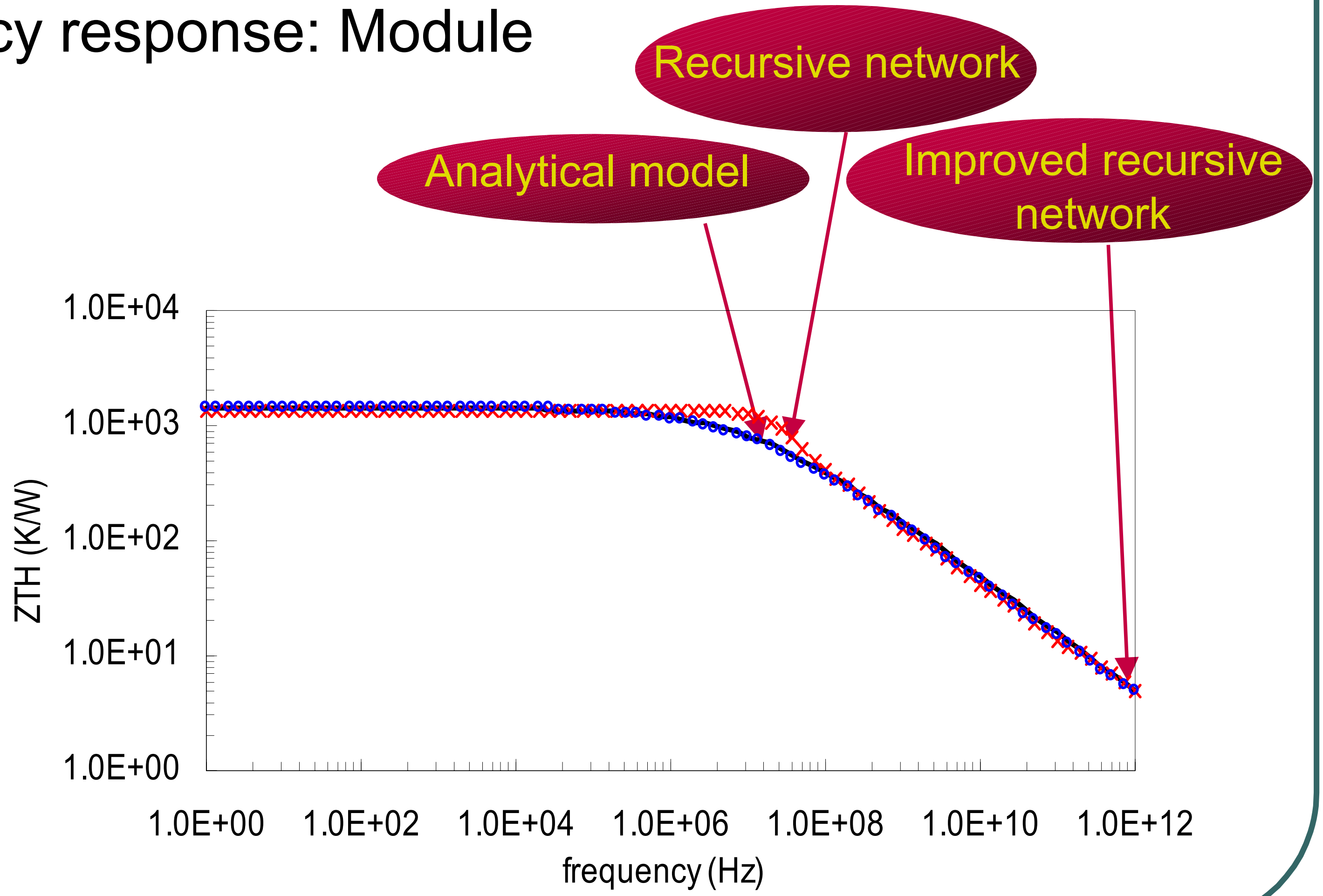

Comparison

- Frequency response: Phase

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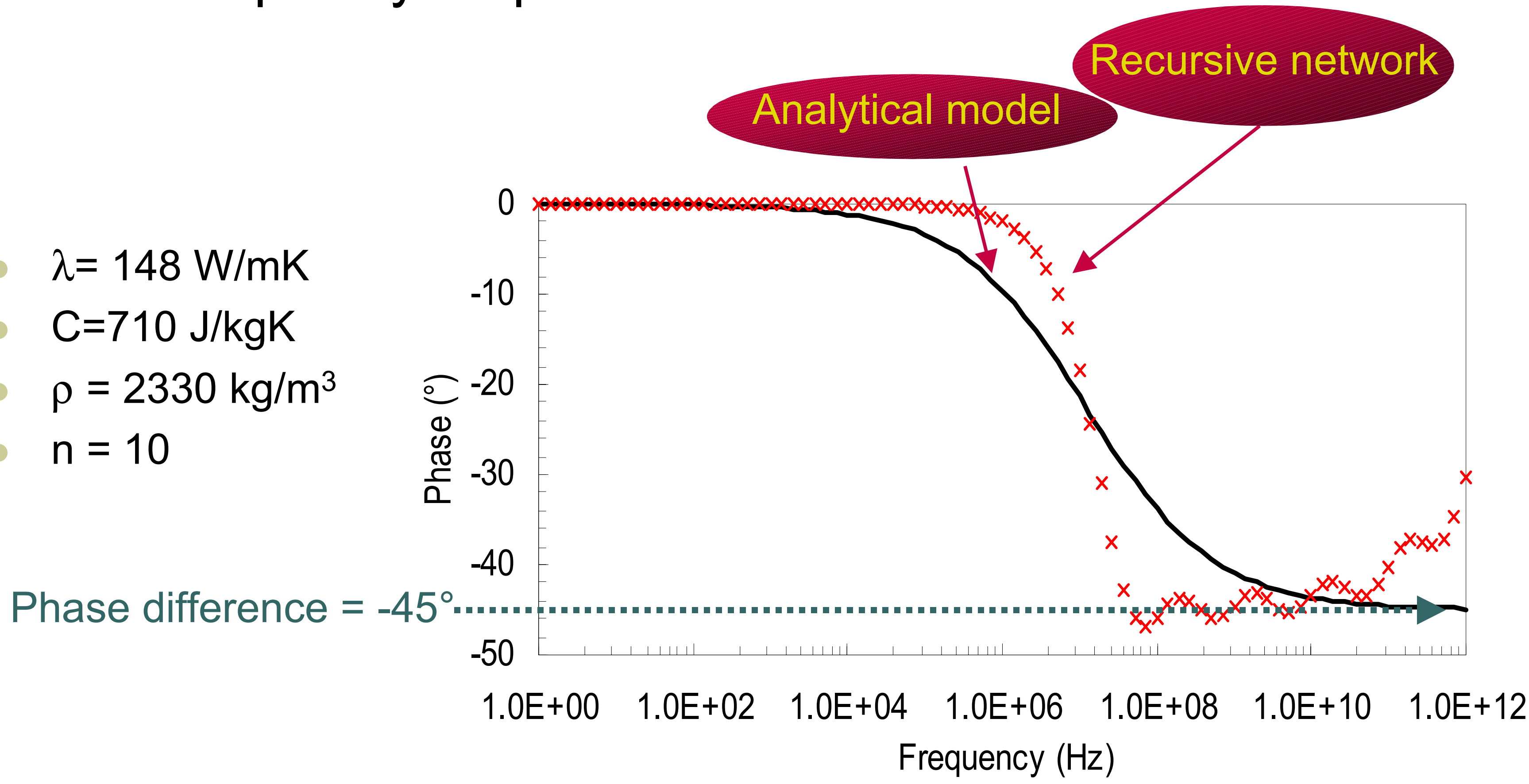
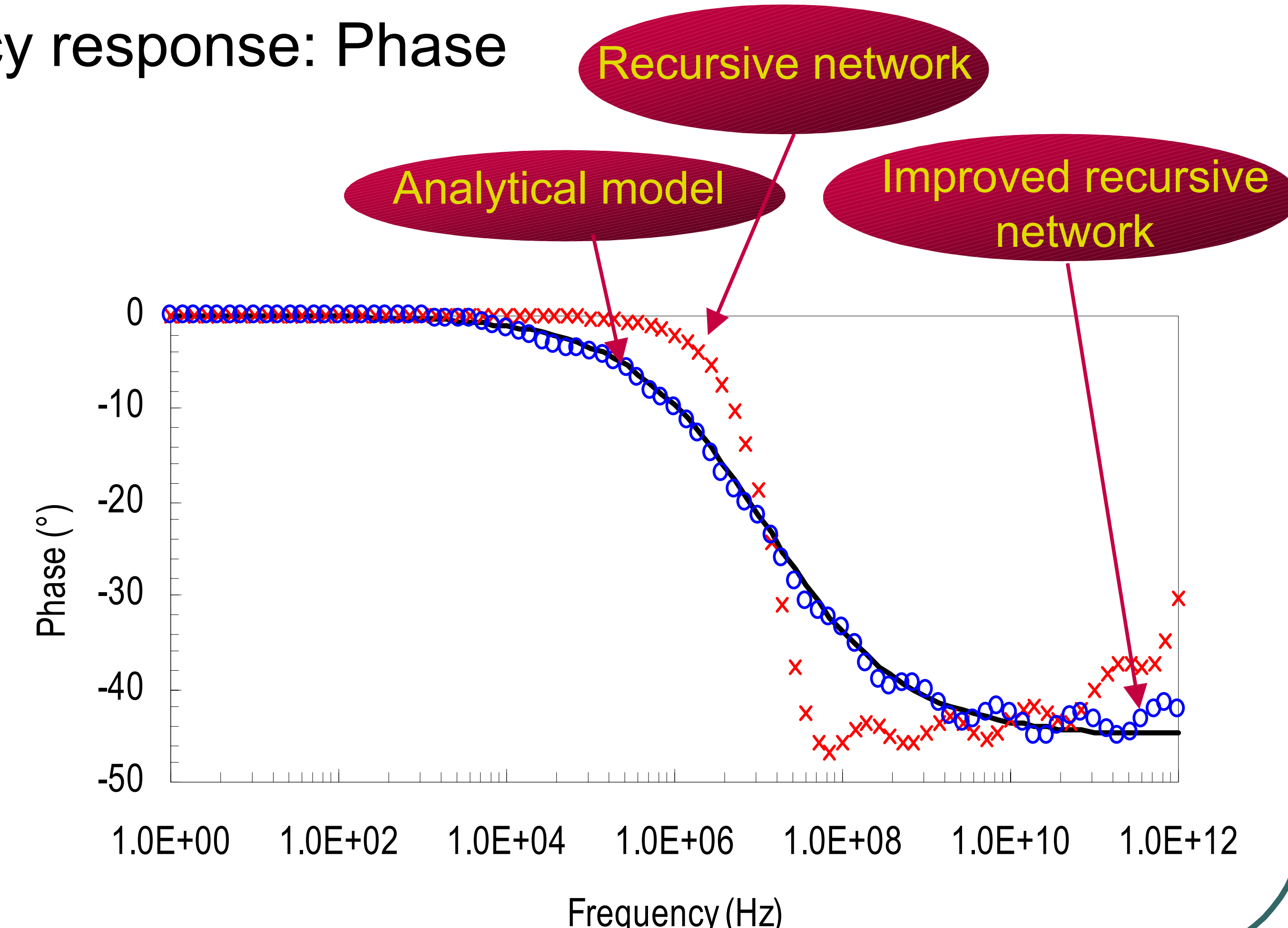


Comparison

- Frequency response: Phase



- λ= 148 W/mK
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Comparison

- Transient response

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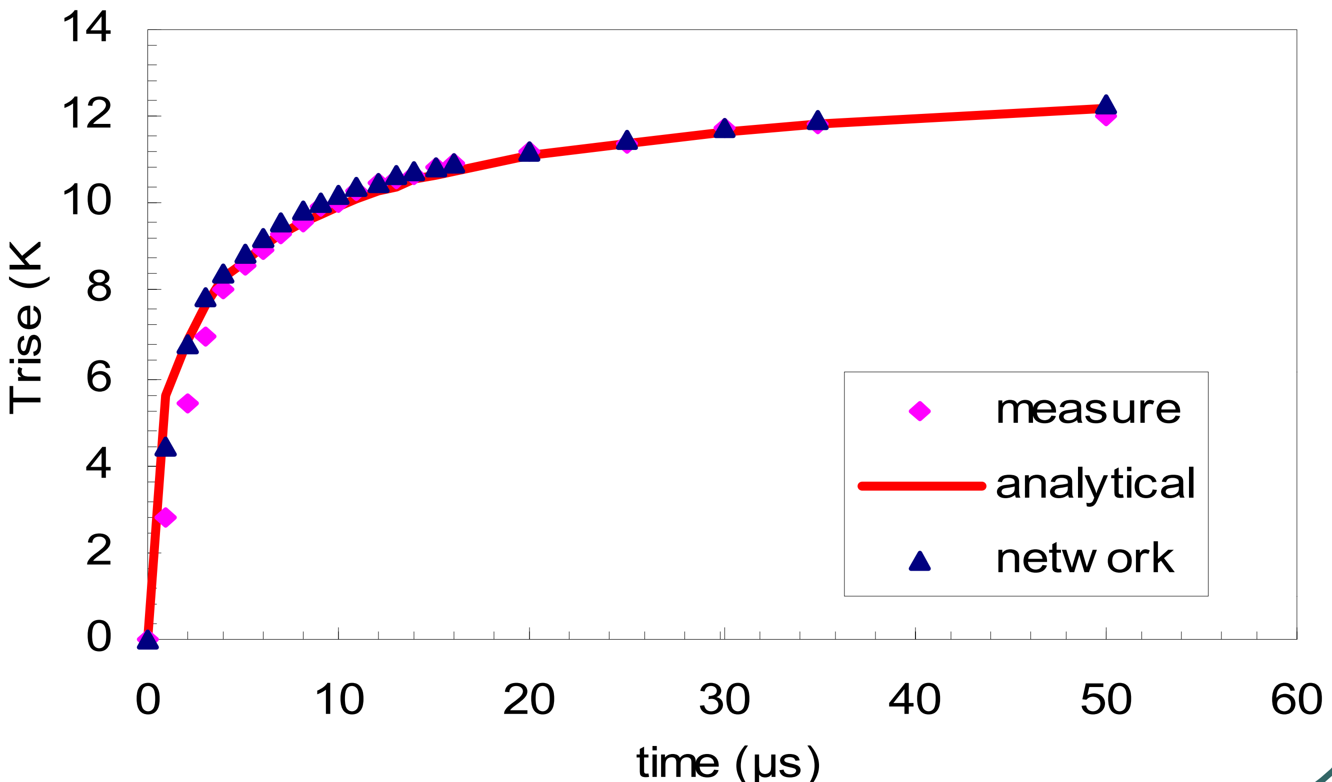
- n = 10



Conclusion

- Development of a temperature dependant dynamic compact model
  - Resolution of heat transfer differential equation
- Equivalent network representation for the SPICE compatibility
  - Linear network
  - Recursive network
  - Improved recursive network
  - → Recursive representation is a good approximation for thermal impedance
  - → Only three parameters to be extracted
- Tested on advanced technology