Analogue realization of fractional differentiation operator

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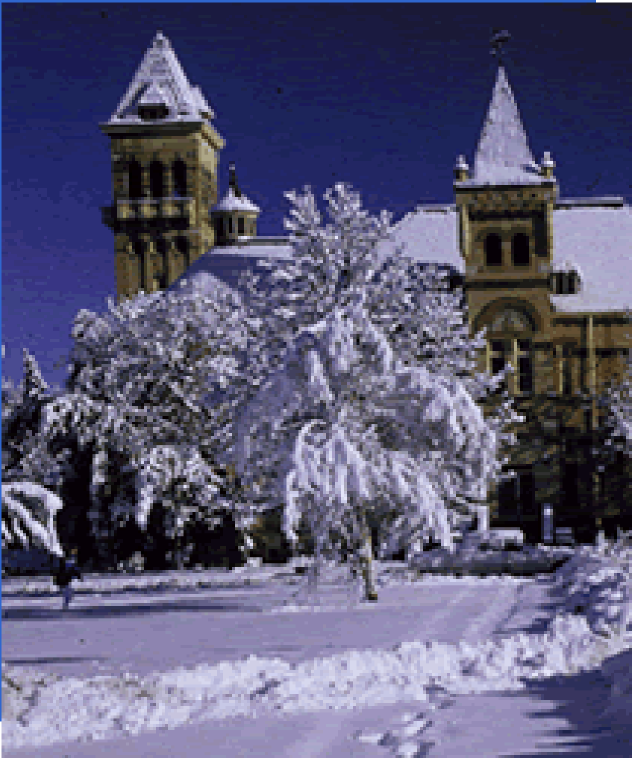
Analogue Realization of Fractional Differentiation operator

- Presentation of Utah State University
- The fractional order controller
- The analogue circuit
- Identification
- Results
- Simulations
- Conclusions

Utah State University

- Since 1862
- Settled in Logan
- 22,000 students
- 45 departments



The Electrical and Computer engineering department

- Since 1929
- Degrees proposed: Bachelor of Science in Computer Eng., Bachelor of Science in Electrical Eng., Concurrent BS/Masters Program, Master of Engineering, Master of Science, Electrical Engineer, Doctor of Philosophy in Electrical Eng.
- 4 research centers :

  - The Center for Self Organizing Intelligent Systems (CSOIS)

  - The National Center for the Design of Molecular Function (NCDMF)

  - Anderson Wireless Center

  - Center for High-speed Information Processing (CHIP)

The CSOIS

- Utah Center of Excellence graduate
- Horizontally-Integrated (multi-disciplinary)
  - – Electrical and Computer Engineering (Home dept.)
  - – Mechanical Engineering
  - – Computer Science
- Vertically-integrated staff (20-40) of faculty, postdocs,engineers, grad students and undergrads

The CSOIS

- Control System Engineering
  - Algorithms (Intelligent Control)
  - Actuators and Sensors
  - Hardware and Software Implementation
- Intelligent Planning and Optimization
- Real-Time Programming
- Electronics Design and Implementation
- Mechanical Engineering Design and Implementation
- System Integration

The Fractional Order Controller

- By knowing the plant, we are supposed to have been able to calculate :
  - Order : m'
  - Gain : C_0
  - Lower bound : f_{min}
  - Upper bound : f_{max}

- The program first calculate :
  - $?_i$
  - $?'_i$
- Which define the frequency response of the controller by the formula :

$$C(j\omega) = C_0 \prod_{i=1}^{m'} \frac{1 + j\omega / \omega'_i}{1 + j\omega / \omega_i}$$

The Fractional Order Controller

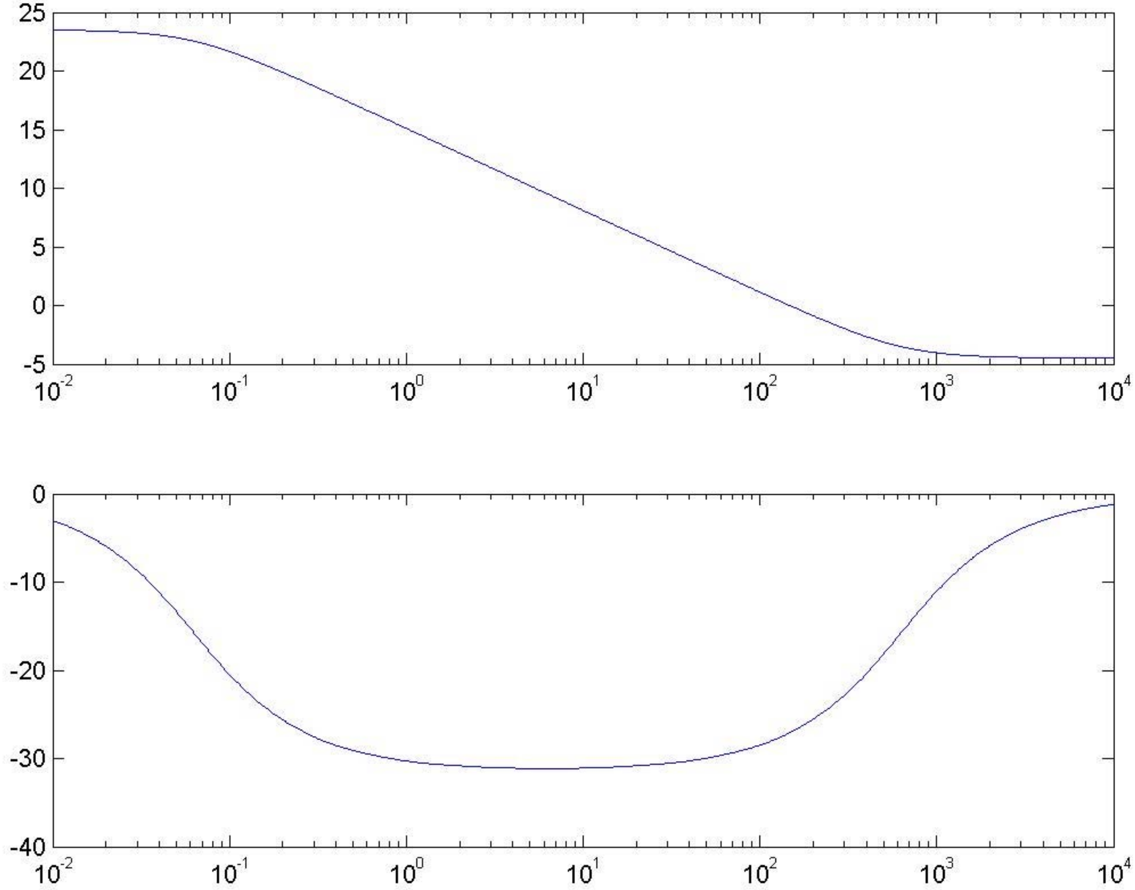


Figure 1 : Bode plot of a fractional order controller

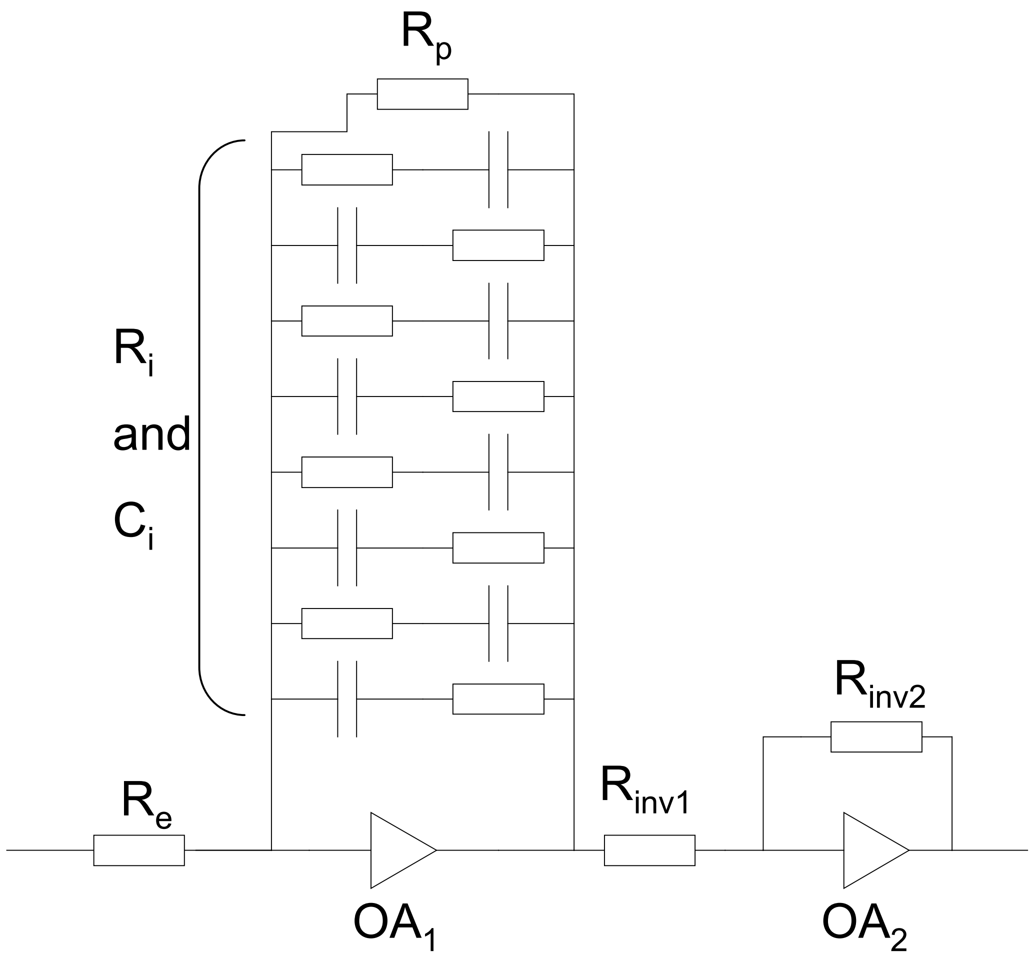
$m' = -0.35$

$C_0 = 15$

$f_{min} = 0.1$ Hz

$f_{max} = 10$ Hz

Analogue Circuit



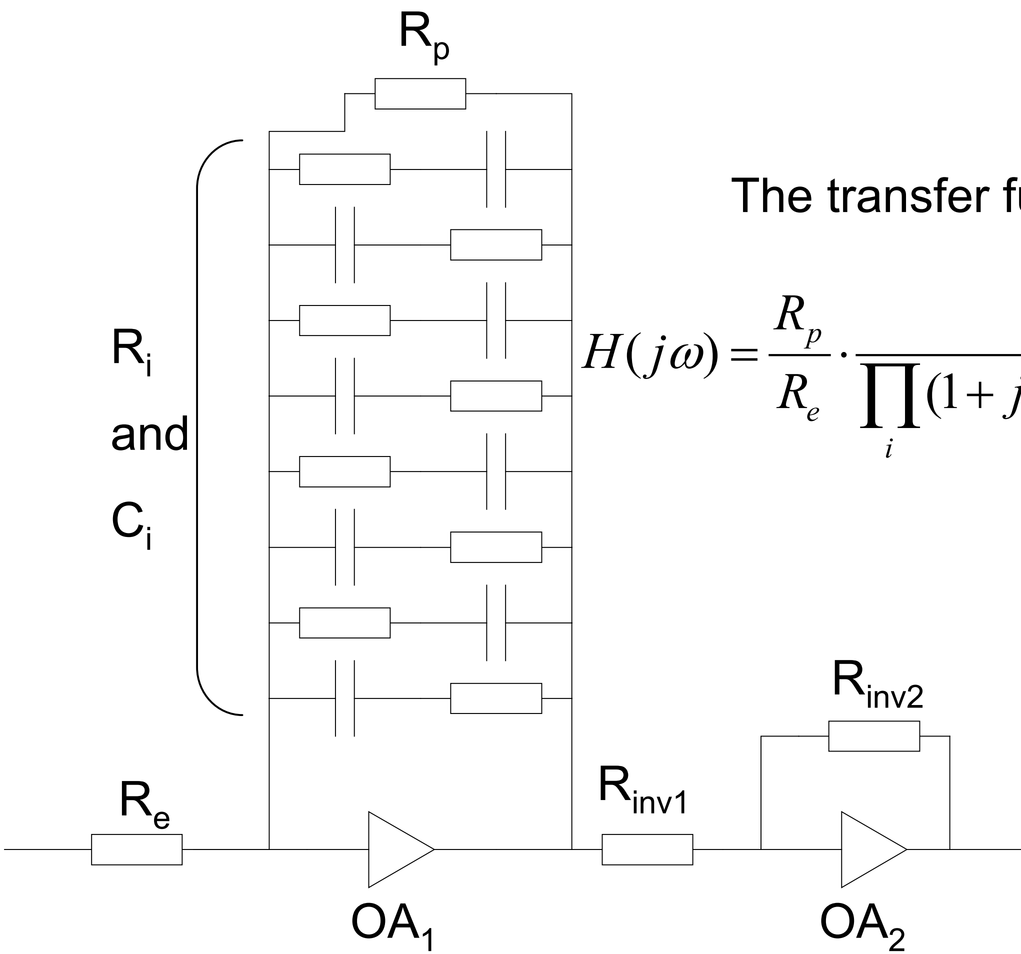
R_e = entrance resistor

R_p = parallel resistor

R_i/C_i = linking impedance

R_{inv1}/R_{inv2} = inverting resitor so that $R_{inv1}=R_{inv2}$

Analogue Circuit



The transfer function of this circuit is :

$$H(j\omega)=\frac{R_p}{R_e}\cdot\frac{\prod_i(1+jR_iC_i\omega)}{\prod_i(1+jR_iC_i\omega)+R_p\sum_i jC_i\omega\prod_{k,k\neq i}(1+jR_kC_k\omega)}$$

Identification

- Now we have two equivalent formulas :

$$C_0 \prod_i^{'} \frac{1 + j\omega / \omega'_i}{1 + j\omega / \omega_i} = \frac{R_p}{R_e} \cdot \frac{\prod_i (1 + jR_i C_i \omega)}{\prod_i (1 + jR_i C_i \omega) + R_p \sum_i jC_i \omega \prod_{k, k \neq i} (1 + jR_k C_k \omega)}$$

- We need to identify each terms of the second formula according to the known values of the terms of the first one.

Identification

- First, we have : $$C_0 = \frac{R_p}{R_e}$$
- Then, we can see that

$$\forall i, \quad \omega'_i \cdot R_i \cdot C_i = 1$$

Identification

- Finally, we have to identify :

$$\prod_i (1 + j\omega / \omega_i) = \prod_i (1 + jR_iC_i\omega) + R_p \sum_i jC_i\omega \prod_{k,k\neq i} (1 + jR_kC_k\omega)$$

- So, we can get 8 equations for 9 tunable parameters. We can tune one and choose :

$$R_p = \frac{-1 \cdot sign(m') \cdot 10^7}{\omega_u}$$

  Which is obtained by an empiric method

Identification

- We can write the equation by this way :

$$\sum_i jC_i\omega\prod_{k\neq i}1+j\frac{\omega}{\omega'_k}=\frac{1}{R_p}\left(\left(\prod_i 1+j\frac{\omega}{\omega_k}\right)-\left(\prod_i 1+j\frac{\omega}{\omega'_k}\right)\right)$$

- We put it in a matrix format :

$$\begin{bmatrix} j\omega\prod_{k\neq 1}1+j\frac{\omega}{\omega'_k} \\ \vdots \\ j\omega\prod_{k\neq m'}1+j\frac{\omega}{\omega'_k}\end{bmatrix}\bullet\begin{bmatrix} C_1 \\ \vdots \\ C_{m'}\end{bmatrix}=\frac{1}{R_p}^{\mathrm{T}}\left[\prod_i\left(1+j\frac{\omega}{\omega'_i}\right)-\prod_i\left(1+j\frac{\omega}{\omega_i}\right)\right]$$

Identification

- We have a system like :

$$A \cdot X = B \Leftrightarrow X = A^{-1} \cdot B$$

- We can now calculate all the values of the circuit using :

$$C_0 = \frac{R_p}{R_e} \quad \text{and} \quad \omega'_i \cdot R_i \cdot C_i = 1$$

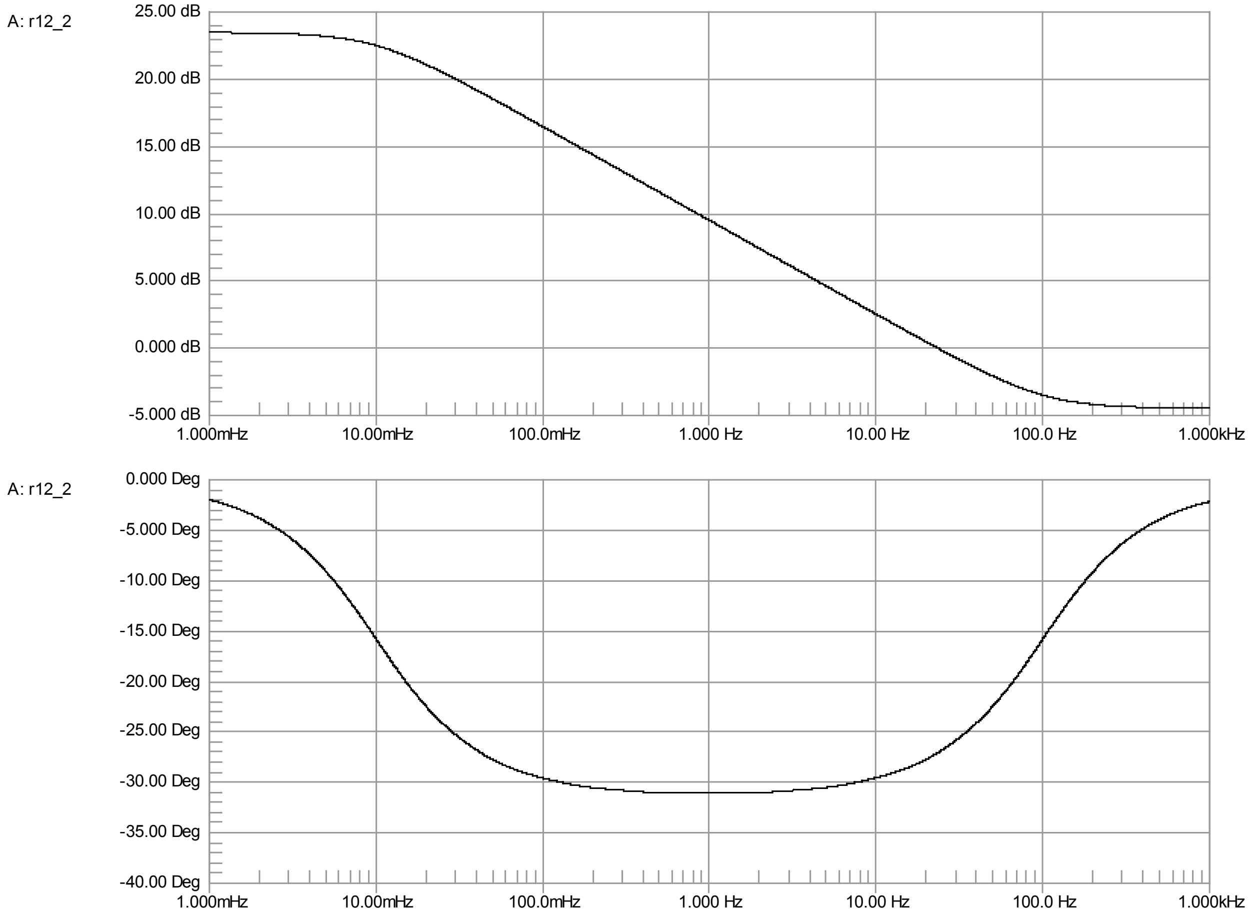
Results

- For m'=0.35, C_0=15, f_{min}=0.1Hz and f_{max}=10Hz, we obtain :

C1	C2	C3	C4	C5	C6	C7	C8
1.62 µF	885 nF	435 nF	208 nF	99.3 nF	47.8 nF	24.0 nF	14.7 nF
R1	R2	R3	R4	R5	R6	R7	R8
4.51 MO	2.61 MO	1.68 MO	1.11 MO	737 kO	484 kO	305 kO	157 kO

Rp	Re
1.59 MO	106 kO

Simulations



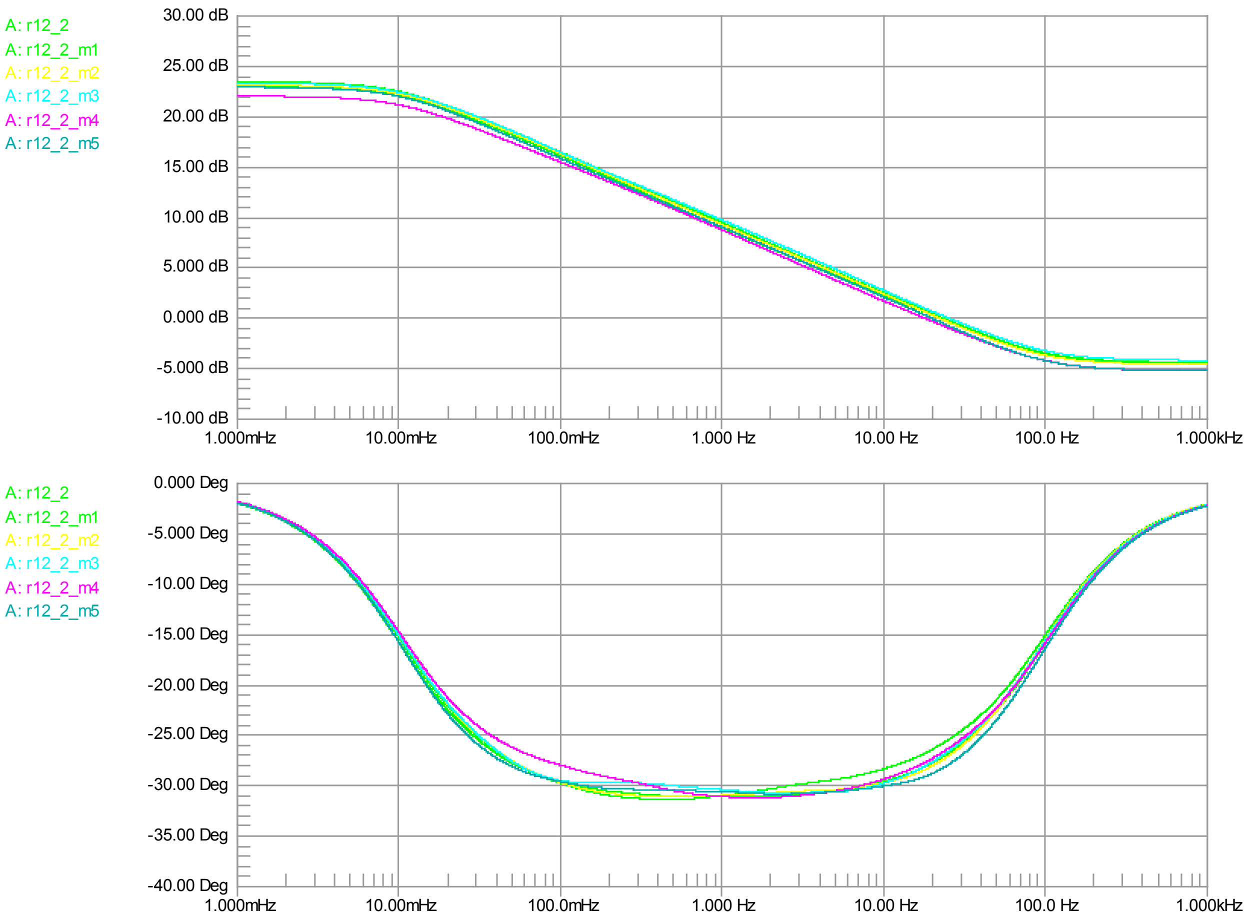
m' = -0.35

C_0 = 15

f_{min} = 0.1 Hz

f_{max} = 10 Hz

Simulations



$m' = -0.35$

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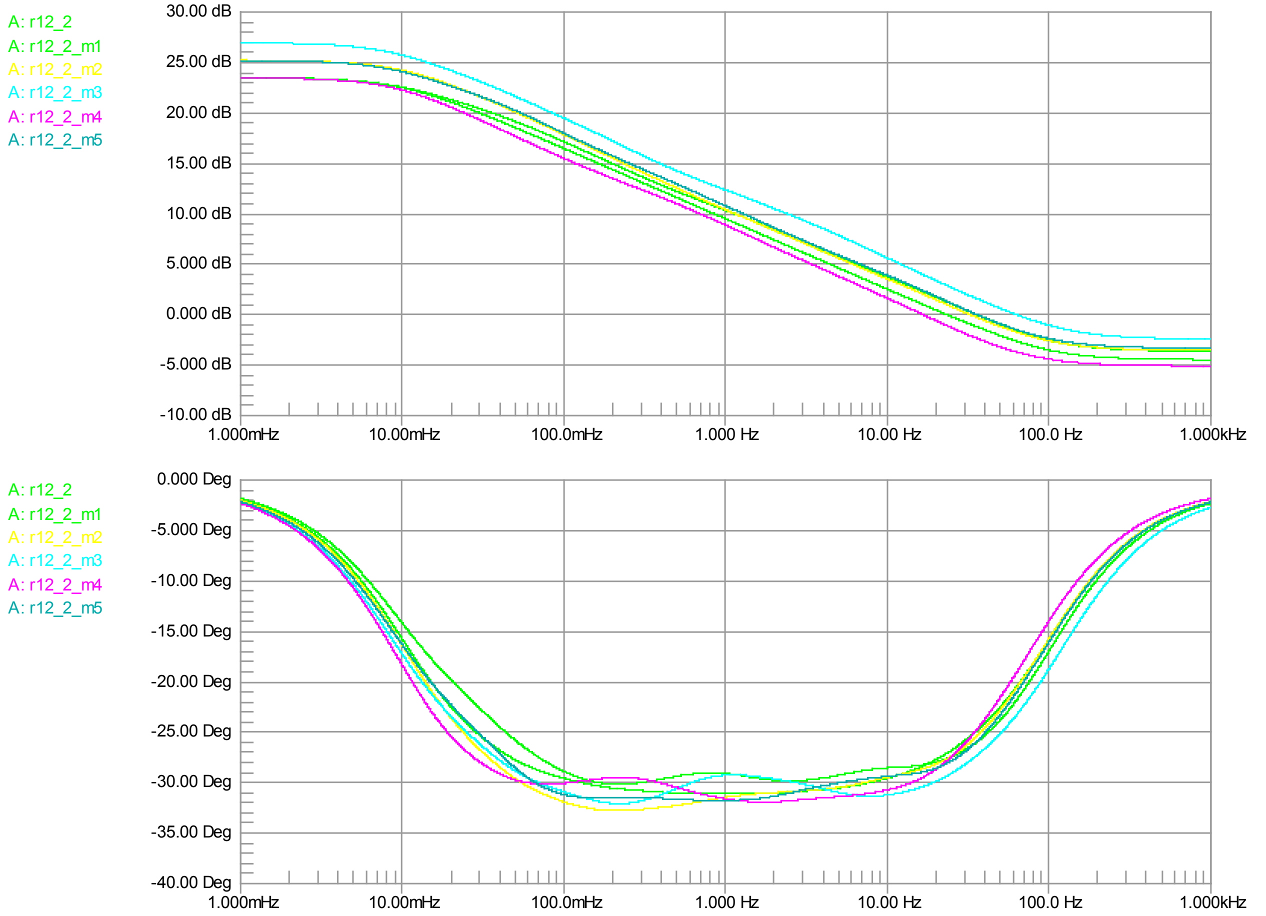
Capacitors and resistors values ±10% gaussian distribution.

Simulations



$m' = -0.35$

$C_0 = 15$

$f_{min} = 0.1$ Hz

$f_{max} = 10$ Hz

Capacitors and resistors values ±10% worst case distribution.

Additional feature

- The program gives you the opportunity to realize a Spice simulation by creating a Spice file of the type :

```
fractional order controller
v1 1 0 dc
e1 3 0 0 2 999k
e2 5 0 0 4 999k
re 1 2 106103.30
rp 2 3 1591549.43
rinv1 3 4 1k
rinv2 4 5 1k
r1 2 6 4510605.10
c1 6 3 0.000001622129
r2 2 7 2613323.53
c2 7 3 0.000000885374
r3 2 8 1683686.31
c3 8 3 0.000000434569
r4 2 9 1111472.41
c4 9 3 0.000000208172
r5 2 10 736654.49
c5 10 3 0.000000099324
r6 2 11 483851.24
c6 11 3 0.000000047820
r7 2 12 305447.38
c7 12 3 0.000000023954
r8 2 13 157277.04
c8 13 3 0.000000014711
.dc v1 0 3.5 0.05
.print dc v(5,0)
.end
```

m’ = -0.35

C_0 = 15

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f_{max} = 10 Hz

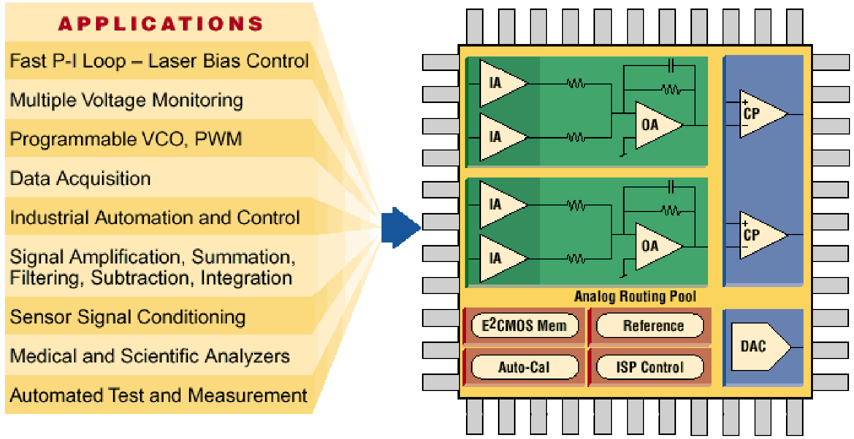
Function of the M-file developed

- Input :
  - Order
  - Gain
  - Lower bound
  - Upper bound
- Output :
  - Values of resistor
  - Values of capacitors
  - Bode plot
  - Step response
  - Impulse response
  - Spice simulation file

If you want this file, send an email at tricaud@enseirb.fr

Possible future realization

- Lattice ispPAC, Integrating in-system programmable analog and digital components on to a single chip.
- For the moment, too few programmable analog components.



ispPAC20 Block Diagram

Bibliography

- « *Les systèmes à dérivées non entières* », A. Oustaloup, 3rd year course, ENSEIRB
- « *US patent nb 5,371,670* » , Dec. 6, 1994, Boris J. Lurie, La Crescenta, Calif.
- « *Analogue Realizations of Fractional Order Controllers* », Petras, I., Podlubny, I., O'Leary, P., Dorcak, L., Vinagre, Faculty BERG, TU Kosice, 2002, p. 84, ISBN 80-7099-627-7.