Action Thématique

Les Systèmes à Dérivées Non Entières : théorie et applications

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Anti-windup system for 1st generation CRONE controller

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Organization of the presentation

1 – Wind-up problem

2 – Common anti-windup systems

3 – 1^{st} generation CRONE control-system design

4 – Frequency domain design of a anti-windup system

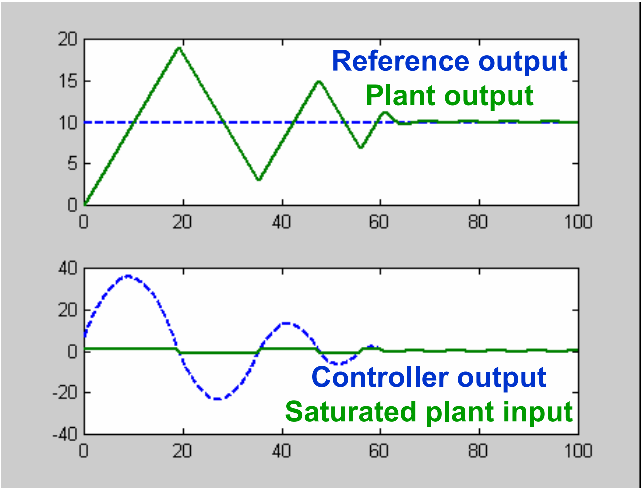
- *Analysis of the windup problem*
- *Modification of the controller to include a windup compensation system*

5 – Fractional controller with windup compensation

- *Continuous-time context*
- *Discrete-time context*

1 – Wind-up problem

• The linear controller of an efficient closed-loop control-system is designed by taking into account (small) disturbance signals
• When the control-input (for instance) of the plant is limited by a saturation, large exogenous signals can lead to poor transient responses



$$\text{Plant}: \frac{1}{s}, \text{Controller}: \frac{1+s}{s\sqrt{2}}$$

$$\text{Saturation limit } -1 \le u_{\text{sat}} \le 1$$

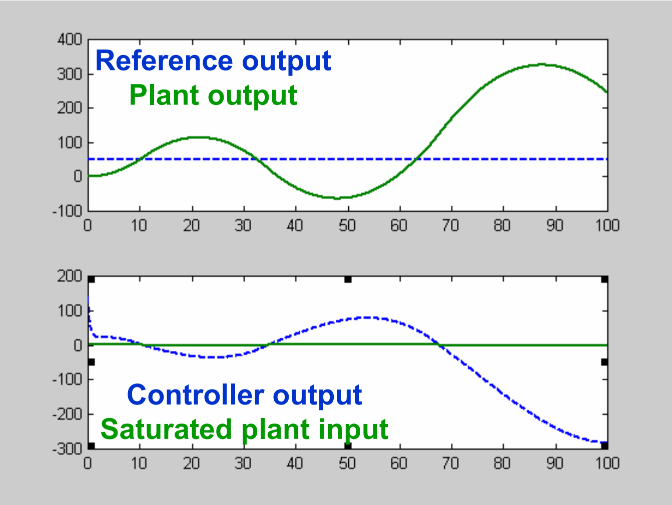
$$\text{Reference output } 10$$

$$(\text{the phase margin is } \pi/4)$$

• Such problems come from the wind-up of the control input provided by an unstable controller (integral action) and/or by a controller with very slow modes

Wind-up problem (2/2)

- The effect of the wind-up problem depends on the plant behaviour also



$$\text{Plant}: \frac{1}{s^2}, \text{Controller}: 2.79\frac{(s+0.357)(s+0.1)}{s(s+2.8)}$$

Saturation limit $-1 \le u_{\text{sat}} \le 1$

Reference output 50

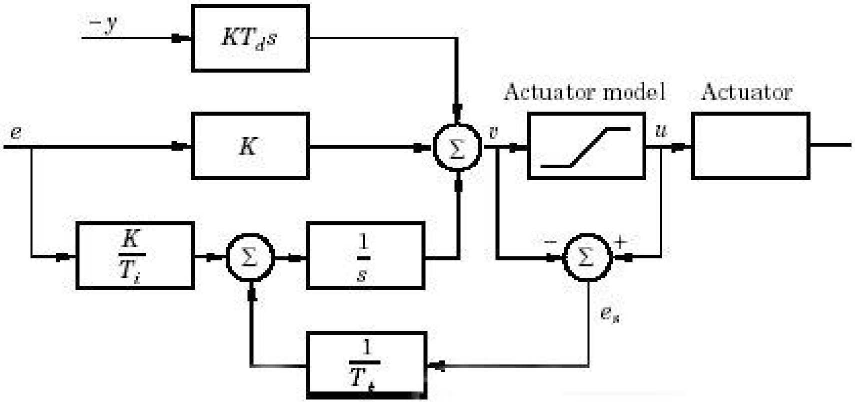
(the phase margin is $\pi/4$)

➔Wind-up problem reduces the stability degree and/or makes unstable the closed-loop system

➔Both the controller and the plant linear behaviours have to be taken into account

2 - Common anti-windup systems

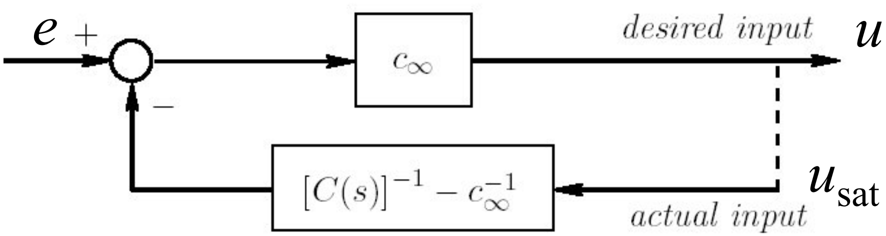
2.1 - Feedback of the integral action (Astrom *et al*)



- Lower is T_t, greater the anti-windup feedback is efficient … but also sensitive to the measurement noise amplified by the derivative action. Common choices: $T_t = T_i$ or $T_t^2 = T_i T_d$
- To be used for a « common » PID controller only

Common anti-windup systems (2/2)

2.2 - Feedback of the HF gain of the controller (Goodwin *et al*)



- $C(s)$ must be bi-proper for $C(s)^{-1} - C_\infty^{-1}$ strictly proper

$$C(s) = \overline{C}(s) + C_\infty^{-1} \text{ with } C_\infty^{-1} = \lim_{s\to\infty} C(s) \neq 0$$

- Equivalent to $u = C\langle e' \rangle$ with $e' = C_\infty^{-1}\left[\mathrm{Sat}\left(\overline{C}\langle e' \rangle + C_\infty e\right) - \overline{C}\langle e' \rangle\right]$

where e' can be considered as a conditioned value of e that takes into account the saturation

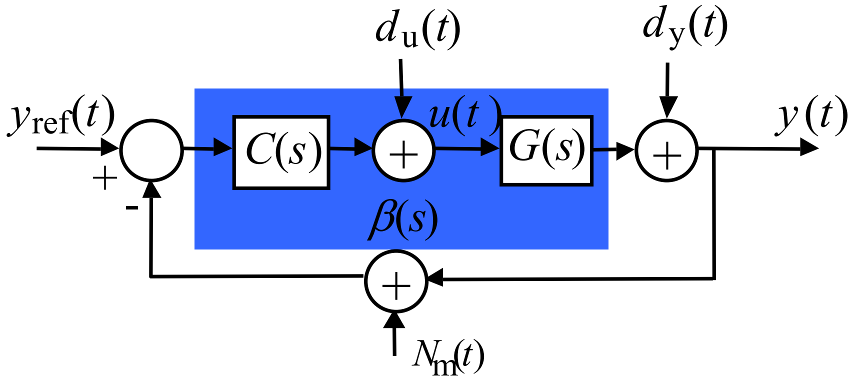
- The whole state of the controller is taken into account
- The behaviour of the controlled plant is not taken into account

3 – 1st generation CRONE control-system design

➡ CRONE is a French acronym which means: fractional order robust control

- Frequency-domain based methodology using fractional differentiation as high-level design parameter (since 1975)
- Continuous or discrete time control of perturbed SISO and MIMO systems
- Use of the common unity-feedback configuration
- Robustness of the stability-degree with respect to the parametric plant perturbation (no over-estimation)
- Avoiding over-estimation of plant perturbation leads to non-conservative robust control-systems and to performance as good as possible
- Control of minimum or non-minimum phase plants, unstable plants or plants with bending modes, time-varying plants, nonlinear plants

Crone control (3 generations)



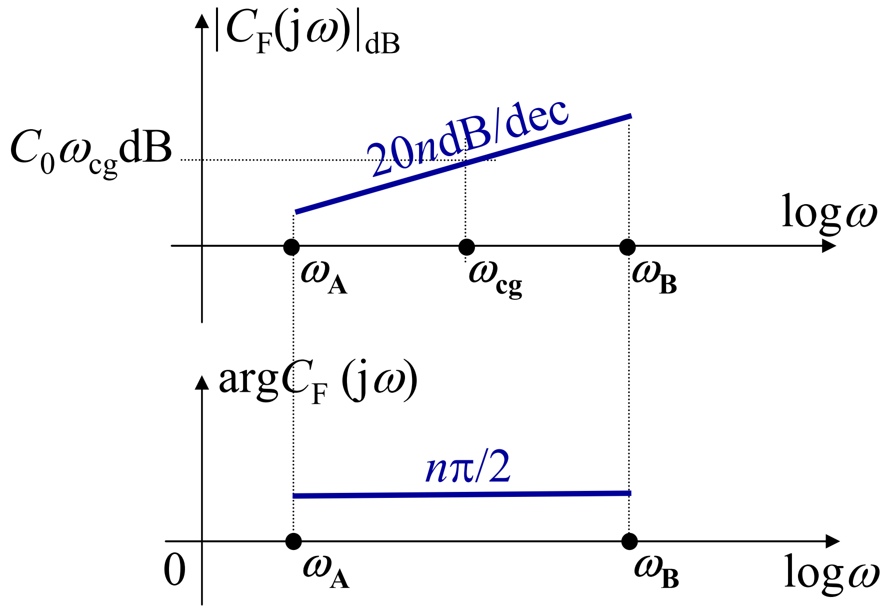
3 CRONE design generations have been developed, successively extending the application fields :

- 1st for gain-like plant perturbation model and for constant plant phase around ω_{cg} (real fractional diff. order for controller definition)
- 2nd for gain-like plant perturbation model (real fractional diff. order for open-loop definition)
- 3rd for most general perturbation model (complex fractional diff. order(s) for open-loop definition)

3.1 - First generation CRONE control

The CRONE controller is defined within a frequency range $[\omega_A, \omega_B]$ around the desired open-loop gain-crossover frequency ω_{cg} from the fractional transfer function of an order n integro-differentiator:

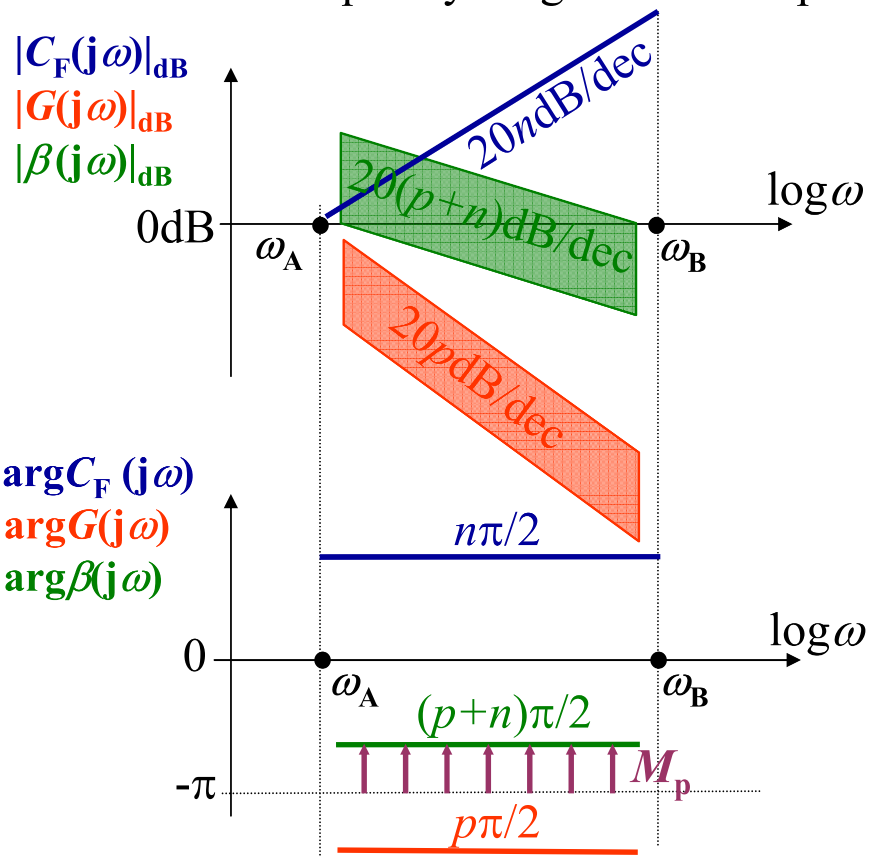
$$C_F(s) = C_0 s^n, \text{ with } n \text{ and } C_0 \in \quad .$$



The constant phase $n\pi/2$ characterizes this controller around frequency ω_{cg}. When frequency ω_{cg} varies, the constant phase controller does not contribute to the phase margin variations.

First generation (robustness)

Particularly appropriate when the desired open-loop gain crossover frequency ω_{cg} is within a frequency range where the plant frequency response is asymptotic.



•Close to ω_{cg}, the plant uncertainty is only gain-like: variation of plant corner-frequencies greatly different from ω_{cg} or/and plant gain variation

•The frequency range $[\omega_A, \omega_B]$ must at least cover the range where frequency ω_{cg} varies

•The phase margin M_ϕ equals $(n+p+2)\pi/2$

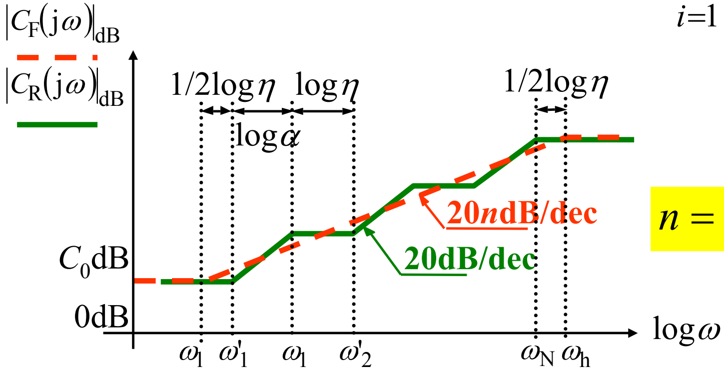
First generation (rational version)

Around ω_{cg}, the initial fractional version $C_F(s)$ of the controller can also be defined by a band-limited transfer function using corner frequencies:

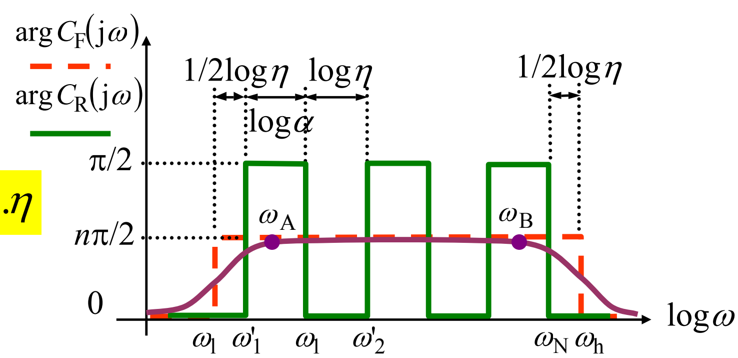
$$C_F(s) = C_0 \left(\frac{1 + s/\omega_l}{1 + s/\omega_h} \right)^n, \text{ with } \omega_l < \omega_A \text{ and } \omega_h > \omega_B.$$

Achievable rational version $C_R(s)$ of the controller, which can be implemented, defined by a transfer function resulting from of a recursive distribution of N cells of real negative zeros and poles:

$$C_R(s) = C_0 \prod_{i=1}^{N} \frac{1 + s/\omega'_i}{1 + s/\omega_i}, \text{ with } N \in {}^{+} \text{ and } \omega'_i, \omega_i \in {}^{+}.$$



$$n = \log \alpha / \log \alpha.\eta$$

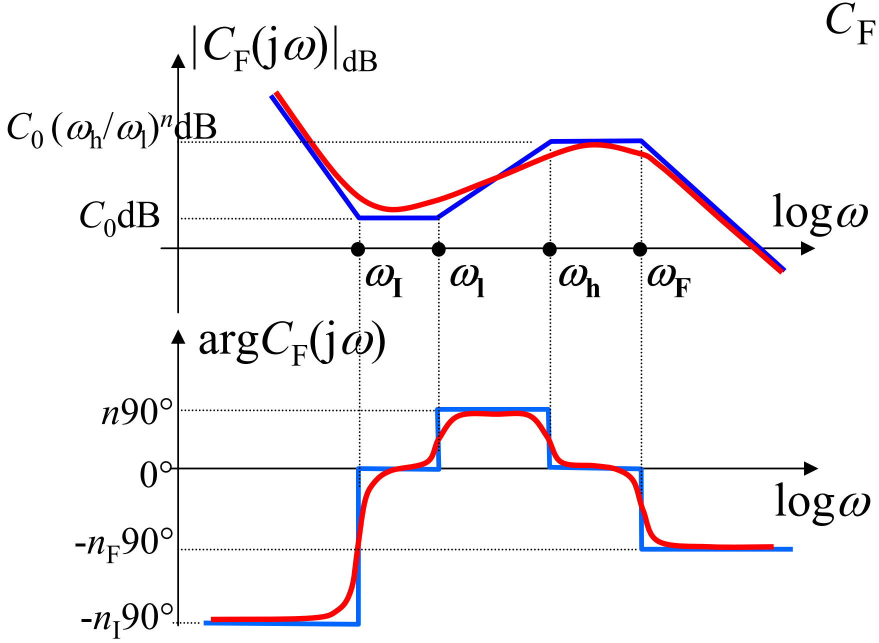


First generation (implementable version)

To manage the control effort level and steady state errors, C_F (and thus C_R) has to be complexified. $C_F(s)$ needs to include:

- an order n_I band-limited integrator
- an order n_F low-pass filter.

$$C_F(s) = C_0\left(\frac{\omega_I}{s}+1\right)^{n_I}\left(\frac{1+s/\omega_l}{1+s/\omega_h}\right)^{n}\frac{1}{(1+s/\omega_F)^{n_F}}$$



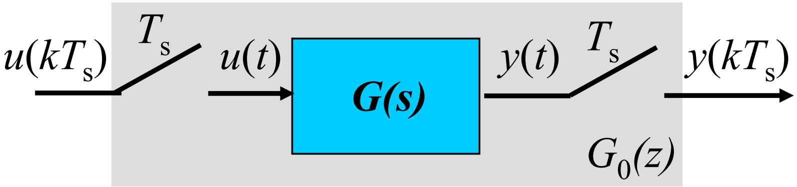
with $n_I, n_F \in {}^+$, and $\omega_I, \omega_F \in {}^+$.

$\omega \in [0, \omega_I]$: **I**ntegral effect
$\omega \in [\omega_I, \omega_l]$: **P**roportional effect
$\omega \in [\omega_l, \omega_h]$: order ***n*** **D**iff. effect
$\omega \in [\omega_h, \omega_F]$: HF gain effect
$\omega \in [\omega_F, \infty[$: low-pass effect

This is a PID^n controller

3.2 - Discrete-time control

A discrete-time control-system design problem with the sampling period T_s is transformed into a pseudo-continuous problem.



Taking into account the zero-order hold effects: $G_0(z) = (1 - z^{-1})\mathcal{Z}\left\{\frac{G(s)}{s}\right\}$

Achieving the bilinear variable change $z^{-1} = \frac{1-w}{1+w}$, $G_0(z)$ becomes $G(w)$.

With $w = jv$ (v is the pseudo-continuous frequency), $v = \tan\left(\frac{\omega T_s}{2}\right)$

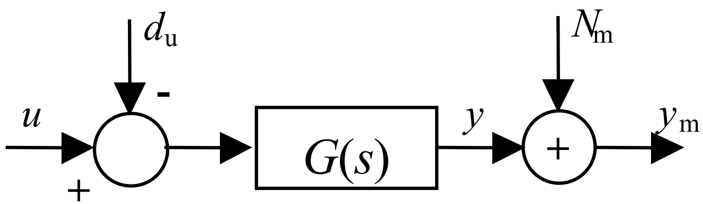
The open-loop $\beta(w)$ and controller $C(w)$ are designed in the pseudo-continuous time domain as in the continuous time domain.

Finally, achieving the inverse variable change $w = \frac{1-z^{-1}}{1+z^{-1}}$, $C(w)$ becomes $C(z)$.

3.3 – Illustrative example

Let a plant modelled by a LTI model $G(s) = \dfrac{y(s)}{u(s)} = \dfrac{\alpha}{s^2}$

$\alpha_0 = 3100$
$1000 \le \alpha \le 10000$



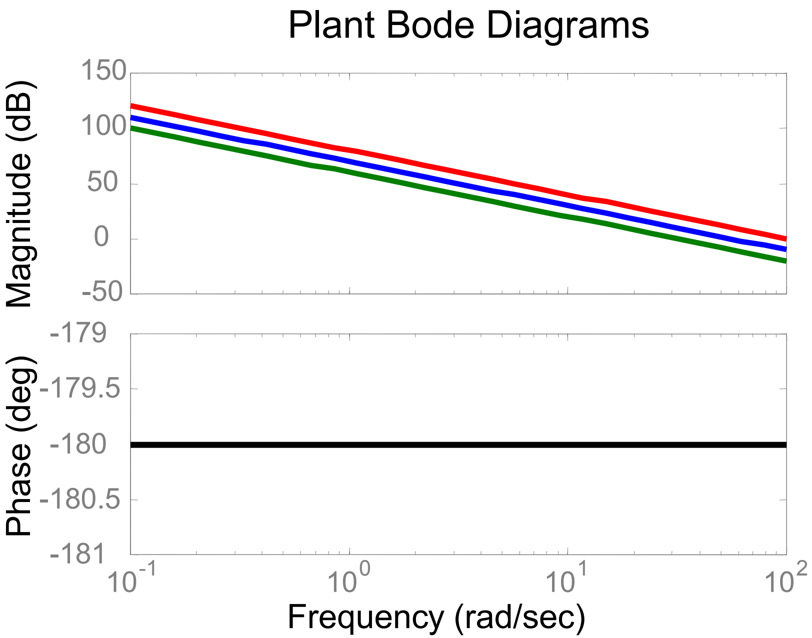
d_u low-frequency disturbance
N_m high-frequency measurement noise

For all α, we want to:

- limit the control-effort u due to the measurement noise ($|u/N_m| \le 20$)
- reduce the steady-state effect of a step disturbance d_u to zero with a settling time as short as possible
- obtain a step response of y to a reference input y_{ref}
  - with a percent overshoot around 20%
  - and a settling time as short as possible

Illustrative example (2/5)

$$G(s) = \frac{\alpha}{s^2} \qquad \alpha_0 = 3100, \quad 1000 \le \alpha \le 10000$$



- A phase margin of 60° leads to an overshoot about 20%.
- The constant open-loop phase around ω_{cg} will equal -130° (-180°+50°=-1.44*90°).
- The constant open-loop gain slope around ω_{cg} will equal -26.6dB/dec.
- As the plant gain variations equals 20dB, the frequency band $[\omega_A, \omega_B]$ needs to cover 0.75 decade at least.
- Then $\omega_B/\omega_A = 5.63$.
- To ensure a robust constant peak magnitude of $T(s)$, ω_B/ω_A is increased to 10

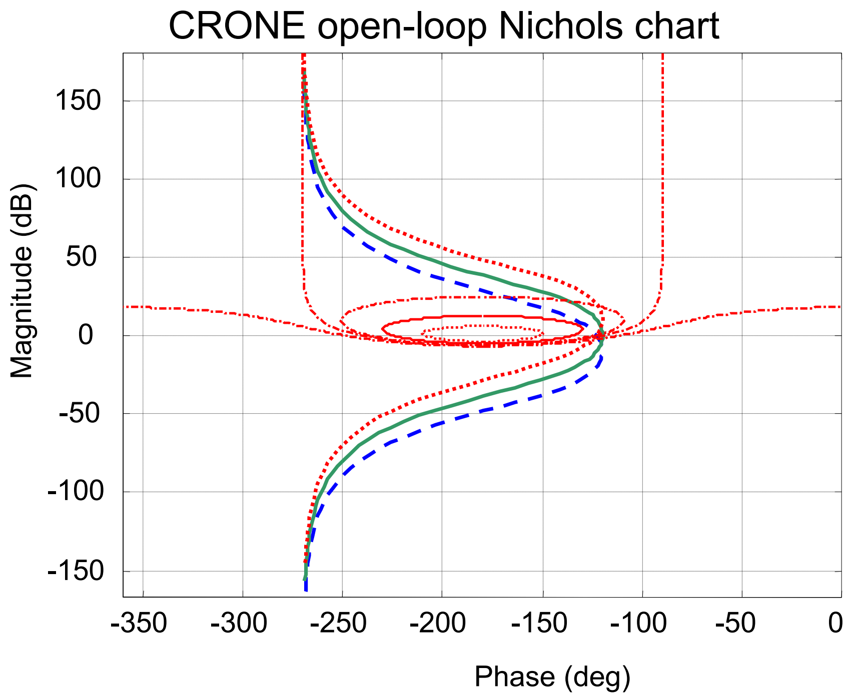
Illustrative example (3/5)

•Taking into account side effects leads to set $\omega_l = \omega_A/10$ and $\omega_h = \omega_B*10$
•To reject the input disturbance $d_u(t)$, $n_I = 1$ and we set $\omega_I = \omega_{cg}/30$
•To avoid the amplification of the high-frequency measurement noise, $n_F = 1$ and we set $\omega_F = \omega_{cg}*30$

•The control effort limitation leads to a nominal value of $\omega_{cg} = 100$ rad/s.

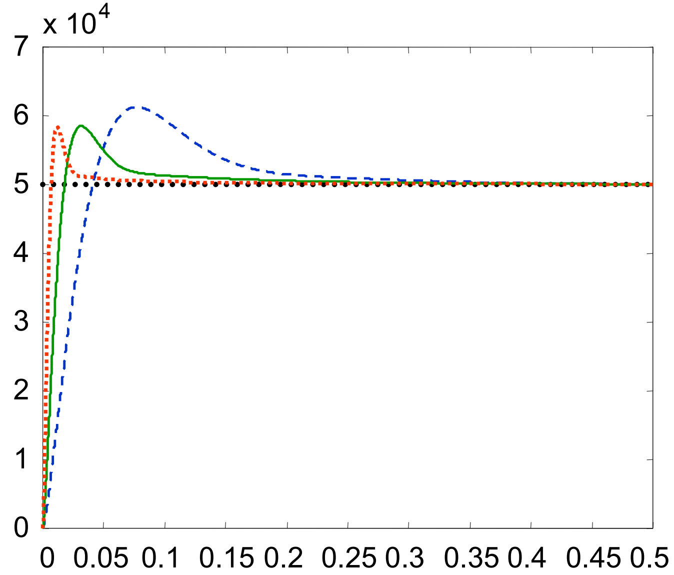
$$C_F(s) = \frac{0.251(3.333+s)(1+s/0.3162)^{0.739}}{s(1+s/3162)^{0.739}(1+s/3000)}$$

Robustness: $57° \leq M_p \leq 60°$



Illustrative example (4/5)

Fractional part (n = 0.739) of the Crone controller synthezised using N = 5 pole-zero cells.

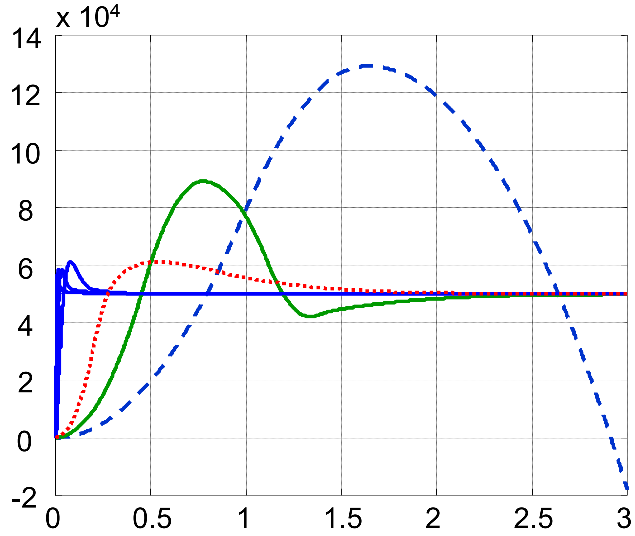


Output step response when the reference input varies from 0 to 5.10^4 for three plant gain value

Overshoot robustness: 17% $\leq O_{\%} \leq$ 22%

Illustrative example (5/5)

The same experiment is achieved with a saturation limit defined by $-200 \leq u_{sat} \leq 200$

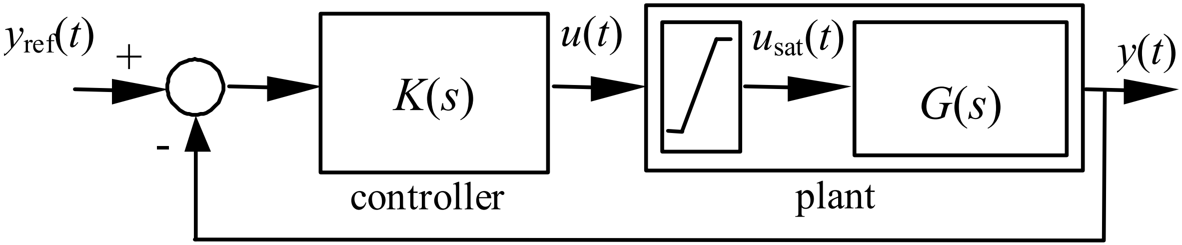


Output step response when the reference input varies from 0 to 5.10⁴ for three plant gain value

The stability degree is reduced (for high values of the plant gain)
or the closed-loop becomes unstable (for low value of the plant gain)

4 – Frequency domain design of a anti-windup system

• The windup problem can be analysed and reduced by using the describing function method.



with
$$\begin{cases} u_{sat}(t) = u(t) & \text{when } |u(t)| \le u_M \\ u_{sat}(t) = u_M & \text{when } u(t) > u_M \\ u_{sat}(t) = -u_M & \text{when } u(t) < -u_M \end{cases}$$

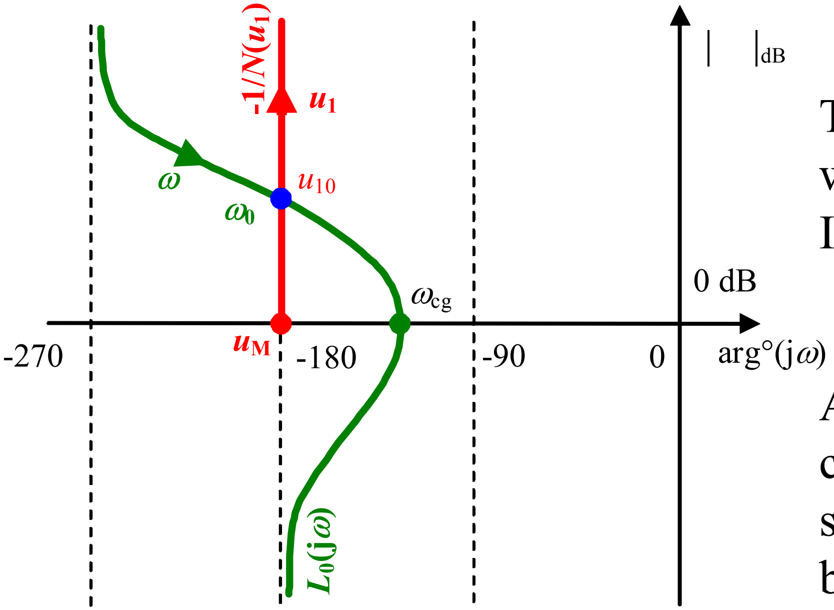
Describing function $N(u_1)$ of the saturation model

$$\begin{cases} N(u_1) = 1 \text{ for } u_1 \le u_M \\ N(u_1) = \dfrac{2}{\pi}\left\{\sin^{-1}\dfrac{u_M}{u_1} + \dfrac{u_M}{u_1}\sqrt{1-\left(\dfrac{u_M}{u_1}\right)^2}\right\} \text{ for } u_1 > u_M \end{cases}$$

4.1 – Analysis of the windup problem

Linear open-loop $L(s) = -\frac{U_{SAT}(s)}{U(s)} = K(s)G(s)$

with an optimal linear (PID type) controller and an inertial plant.

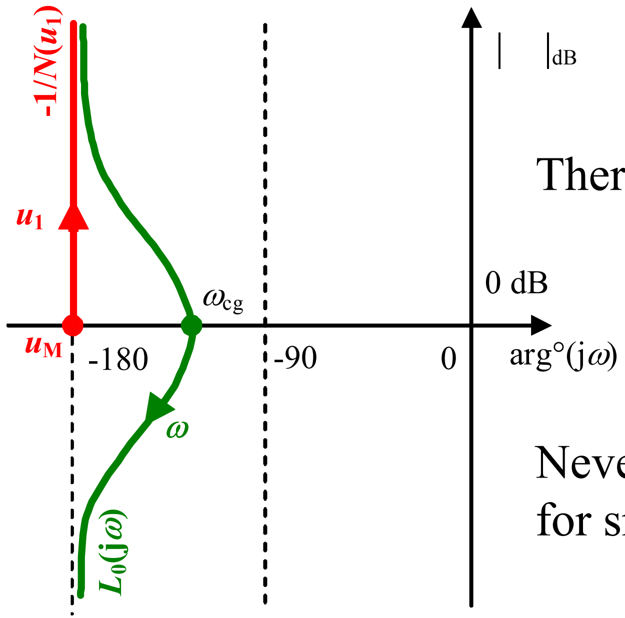


The closed loop system may be unstable when $u(t) > u_{10}$.
It needs to increase $|K(j\omega_0)|$ to increase u_{10}.

As the controller is the linear optimal controller that meets the (linear) specification, the modification of its behaviour for small signals is undesirable

Analysis of the windup problem (2/2)

What's happen if there are an integrating plant and a PID type controller ?



There is no crossing point defined by $L(j\omega_0) = -1/N(u_1)$.

Nevertheless $L(j\omega_0)$ and $-1/N(u_1)$ are very close for small ω and large u_1.

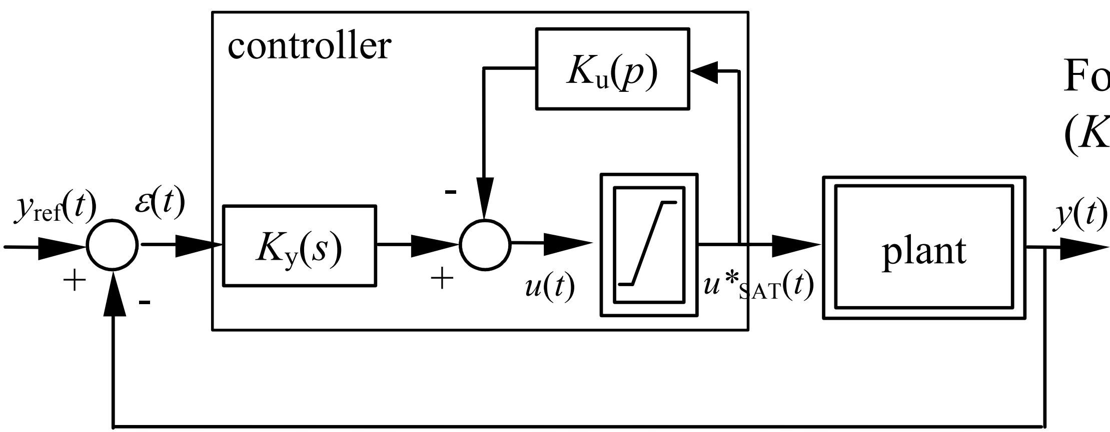
➔ Very low-damped transient responses can be predicted for large value of $u(t)$.

4.3 - Modification of the controller to include a windup compensation system

Taking into account the model of the plant nonlinearity, many authors have proposed to split the optimal controller $K(s)$ so that:

- the linear behaviour of the new controller remains the same than before
- the output of the controller tracks its saturated value.

Taking into account the linear model of the plant also, Ygorra *et al.* proposed a frequency-domain design method to split the controller.

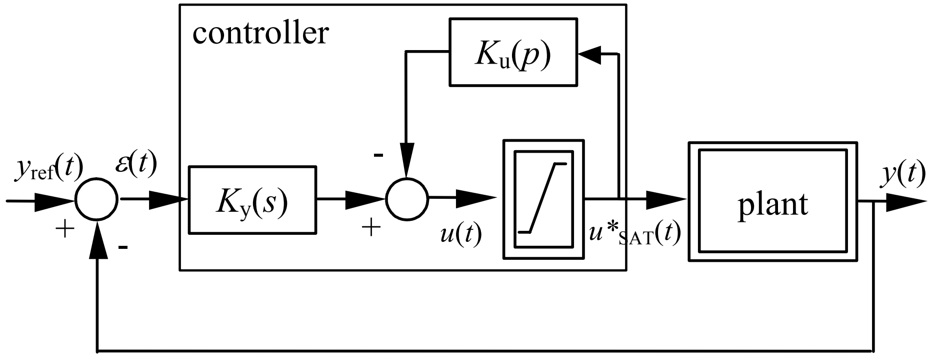


An inner loop feedbacks a part of the controller

For small signals ($|u(t)| \leq u_M$), (K_y, K_u) must ensures that:

$$\frac{U(s)}{\varepsilon(s)} = \frac{K_y(s)}{1 + K_u(s)} = K(s)$$

Windup compensation system (2/4)



$$K(s)=\frac{N_K(s)}{D_K(s)}\frac{D(s)}{D(s)},$$

$$K_y(s)=\frac{N_y(s)}{\Delta(s)} \text{ and } K_u(s)=\frac{N_u(s)}{\Delta(s)}$$

N_K, N_y, N_u, D_K and Δ are the polynomials that define the transfer functions of K, K_y and K_u, and where the polynomial D is with left-half plane zeros

$$\frac{N_y(s)}{\Delta(s)+N_u(s)}=\frac{N_K(s)}{D_K(s)}\frac{D(s)}{D(s)}\Rightarrow\begin{cases}N_u(s)=D_K(s)D(s)-\Delta(s)\\ N_y(s)=N_K(s)D(s)\end{cases}$$

The linear part of the open-loop transfer function L_{NL} is now

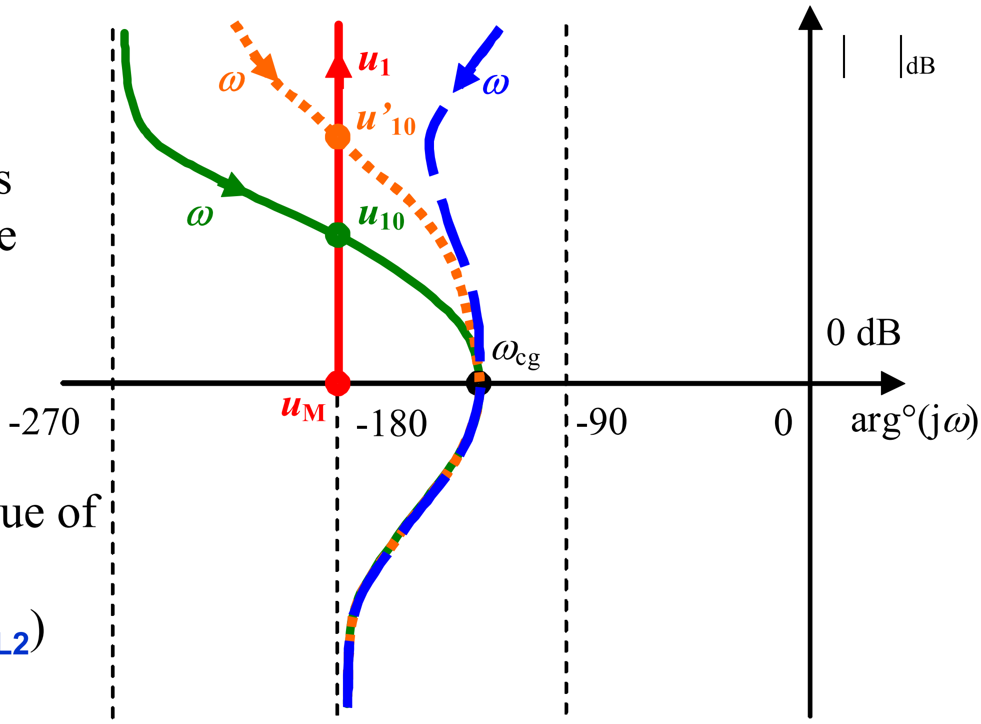
$$L_{NL}(s)=-\frac{U(s)}{U_{SAT}(s)}=-1+\frac{D_k(s)D(s)}{\Delta(s)}(1+L(s))$$

Windup compensation system (3/4)

At low frequency ($\omega \ll \omega_{cg} \Rightarrow |L(j\omega)| \gg 1$)

$$L_{NL}(s) \approx \frac{D_k(s)D(s)}{\Delta(s)} L(s)$$

The transfer function $D_K D/\Delta$ looks like a compensator that permits the modification of the open-loop.



It can be possible to increase the value of u_{10} (***L_{0NL1}***) and sometimes to make disappear the stability problem (***L_{0NL2}***)

Windup compensation system (4/4)

As the open loop $L_{NL}(j\omega)$ equals $L(j\omega)$ at high frequency ($\omega > \omega_{cg}$), D and Δ are such that:

$$\lim_{\omega \to \infty} D_k(j\omega) D(j\omega) = \lim_{\omega \to \infty} \Delta(j\omega)$$

Thus it is useful that Δ includes the high frequency part of D_K (ie. all the zeroes whose modulus are greater than ω_{cg}). The other parts of Δ and D are determined to:

•shape well the frequency response of the perturbed open-loop L_{NL}

•ensure the degree condition:

$$\text{degree} D_k(s) + \text{degree} D(s) = \text{degree } \Delta(s)$$

Then polynomials N_u and N_y can be determined from polynomials D and Δ.

5 - Fractional controller with windup compensation

Let be $K(s)$ an optimal linear controller designed with the first generation Crone design method

$$K(s)=\frac{C_0\left(\omega_{\mathrm{I}}+s\right)^{n_{\mathrm{I}}}\left(1+s/\omega_{\mathrm{l}}\right)^{n}}{s^{n_{\mathrm{I}}}\left(1+s/\omega_{\mathrm{h}}\right)^{n}\left(1+s/\omega_{\mathrm{F}}\right)^{n_{\mathrm{F}}}}$$

As the fractional part should need to be approximated with N poles (and zeros), the final denominator degree of the rational filter K would be high. Thus it would lead to high degree of polynomial Δ and then to high order of filters K_{u} and K_{y}.

To avoid that, it is possible to use the previous windup compensation method while taking and account the fractional feature of K.

5.1 - Continuous-time context

The CRONE controller is written

$$K(s) = \frac{N_{\mathrm{K}}(s)}{D_{\mathrm{Kl}}(s) D_{\mathrm{Kh}}(s)} \quad \text{with} \quad \begin{cases} N_{\mathrm{K}}(s) = C_0 (\omega_{\mathrm{I}} + s)^{n_{\mathrm{I}}} (1 + s/\omega_{\mathrm{l}})^{n} \\ D_{\mathrm{Kl}}(s) = s^{n_{\mathrm{I}}} \\ D_{\mathrm{Kh}}(s) = (1 + s/\omega_{\mathrm{h}})^{n} (1 + s/\omega_{\mathrm{F}})^{n_{\mathrm{F}}} \end{cases}$$

where N_{K} is the numerator of K and where D_{Kl} and D_{Kh} are the low and high frequency parts of its denominator ($\omega_{\mathrm{h}} > \omega_{\mathrm{cg}}$).

As Δ includes the high frequency part of D_{K}, the "compensator" $D_{\mathrm{K}} D/\Delta$ is

$$\frac{D_{\mathrm{k}}(s) D(s)}{\Delta(s)} = \frac{D_{\mathrm{Kl}}(s) D_{\mathrm{Kh}}(s) D(s)}{D_{\mathrm{Kh}}(s) \Delta^*(s)} = \frac{D_{\mathrm{Kl}}(s) D(s)}{\Delta^*(s)}$$

where Δ^* is the complementary part of Δ, and where D is an degree n_{D} polynomial with

$$n_{\mathrm{I}} + n_{\mathrm{D}} = \text{degree } \Delta^*(s)$$

5.1 - Continuous-time context (2/3)

Polynomials D and Δ^* (used to shape the "nonlinear" open-loop L_{NL} at low frequency) are chosen as products of first order polynomials with unit static gains :

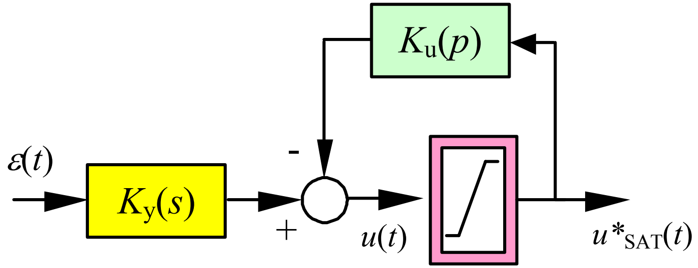
$$D(s)=\prod_{i=1}^{n_D}\left(1+\frac{s}{\omega_{Di}}\right) \text{ and } \Delta^*(s)=\prod_{i=1}^{n_I+n_D}\left(1+\frac{s}{\omega_{\Delta i}}\right)$$

Remark: If n_I equals 1, taking a polynomial D with a null degree would lead to a degree 1 of Δ^* (one degree of freedom) and thus does not provide enough degree of freedom as the "compensator" $D_K D/\Delta$ needs also to be with a unit gain at high frequency.

$$\text{As}\begin{cases}N_u(s)=D_K(s)D(s)-\Delta(s)\\ N_y(s)=N_K(s)D(s)\end{cases}, \text{ then }\begin{cases}N_u(s)=(1+s/\omega_h)^n(1+s/\omega_F)^{n_F}\left[s^{n_I}D(s)-\Delta^*(s)\right]\\ N_y(s)=C_0(\omega_I+s)^{n_I}(1+s/\omega_1)^n D(s)\end{cases}$$

5.1 - Continuous-time context (2/3)

$$K_u(s) = \frac{N_u(s)}{D_{Kh}(s)\Delta^*(s)} = \frac{s^{n_I}\prod_{i=1}^{n_D}\left(1+\frac{s}{\omega_{Di}}\right) - \prod_{i=1}^{n_I+n_D}\left(1+\frac{s}{\omega_{\Delta i}}\right)}{\prod_{i=1}^{n_I+n_D}\left(1+\frac{s}{\omega_{\Delta i}}\right)}$$



$$K_y(s) = \frac{N_y(s)}{D_{Kh}(s)\Delta^*(s)} = \frac{C_0(\omega_I + s)^{n_I}}{(1+s/\omega_F)^{n_F}}\left(\frac{1+s/\omega_l}{1+s/\omega_h}\right)^n \frac{\prod_{i=1}^{n_D}\left(1+\frac{s}{\omega_{Di}}\right)}{\prod_{i=1}^{n_I+n_D}\left(1+\frac{s}{\omega_{\Delta i}}\right)}$$

K_u is a very low order filter and order of filter K_y close to that of K

➔Complexity of the final controller close to that of the initial linear controller K.

5.2 - Discret-time context

The w-domain fractional transfer function of controller K can be splitted into $K_u(w)$ and $K_y(w)$.

To ensure that the open loop $L_{NL}(jv)$ equals $L(jv)$ at high frequency ($v > v_{cg}$), D and Δ should be such that:

$$\lim_{v\to\infty} D_k(jv)D(jv) = \lim_{v\to\infty} \Delta(jv)$$

Nevertheless, the discrete-time filter $K_u(z)$ needs to be to be achievable and thus strictly proper. It can be if

$$\lim_{z\to\infty} K_u(z) = \lim_{z\to\infty} \frac{D_k(z)D(z)}{\Delta(z)} - 1 = 0$$

As $w = (z-1)/(z+1)$, the high frequency constraint is

$$\lim_{w\to 1} \frac{D_k(w)D(w)}{\Delta(w)} = 1$$

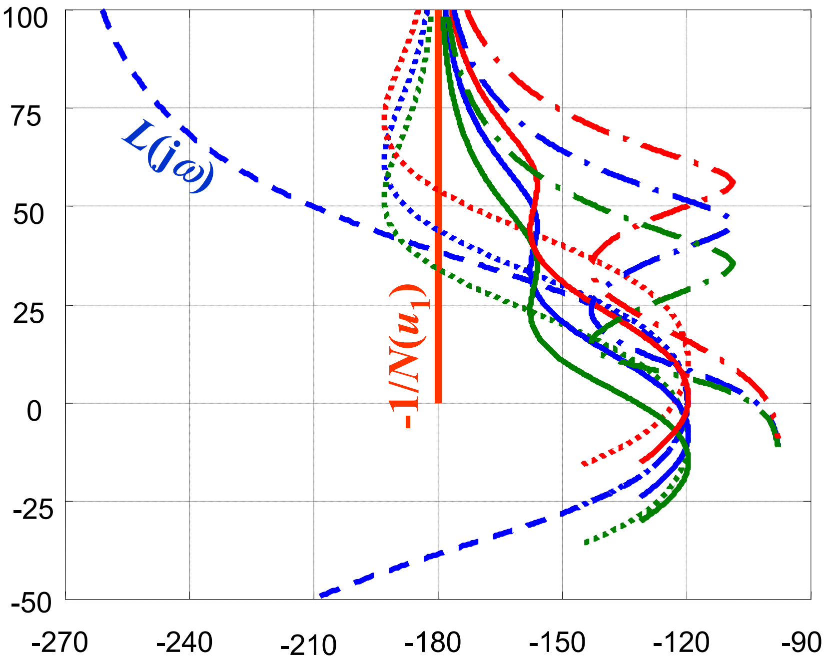
5.3 - Illustrative example

To split the controller we choose the polynomials D and Δ^*

$$D(s) = \left(1 + \frac{s}{\omega_{D1}}\right) \text{ and } \Delta^*(s) = \left(1 + \frac{s}{\omega_{\Delta 1}}\right)\left(1 + \frac{s}{\omega_{\Delta 2}}\right)$$

As $D_{K1}D/\Delta^*$ needs to be with a unit gain at high frequency and as $D_{K1}(s) = s$, then $\omega_{D1} = \omega_{\Delta 1}\ \omega_{\Delta 2}$.

We set $\omega_{\Delta 1} = \omega_{\Delta 2} = \sqrt{\omega_{D1}}$.



L_{NL} for plant gain $\alpha \in \{1000, 3100, 1000\}$ and ω_{D1}: 3rad/s (······),30 rad/s (——) and 300 rad/s (–•–•–)

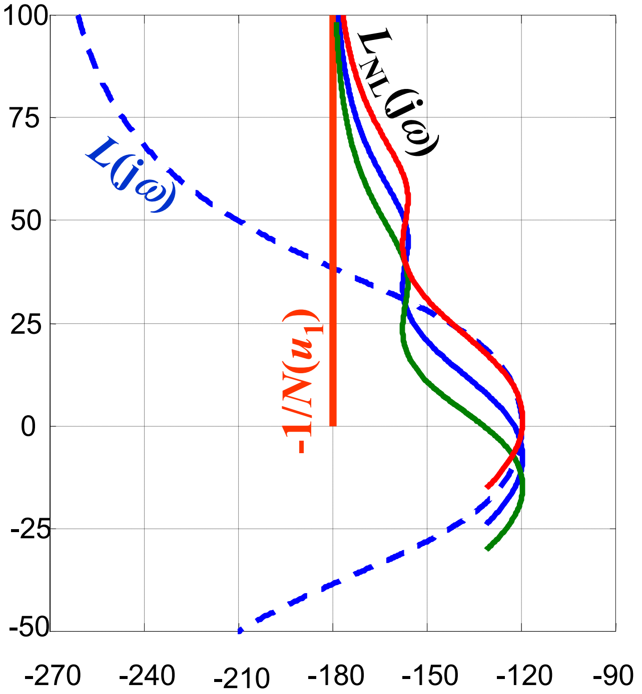
Illustrative example (2/4)

As the corner frequency $\omega_{\Delta 1}$ starts the roll-off of K_y, it manages the peak magnitude peak of K_y. Thus, to limit the amplification of the measurement noise, it is very important to choose the lowest value of ω_{D1} as possible.

ω_{D1} = 30 rad/s provides a sufficient robust distance from the perturbed open-loop to $-1/N(u_1)$.

$$K_u(s) = \frac{19.05s - 30}{s^2 + 10.95s + 30}$$

$$K_y(s) = \frac{0.251(3.333 + s)}{1 + s/3000} \left(\frac{1 + s/0.3162}{1 + s/3162} \right)^{0.739} \frac{1 + s/30}{(1 + s/5.48)^2}$$

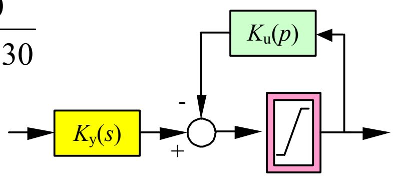


Illustrative example (3/4)

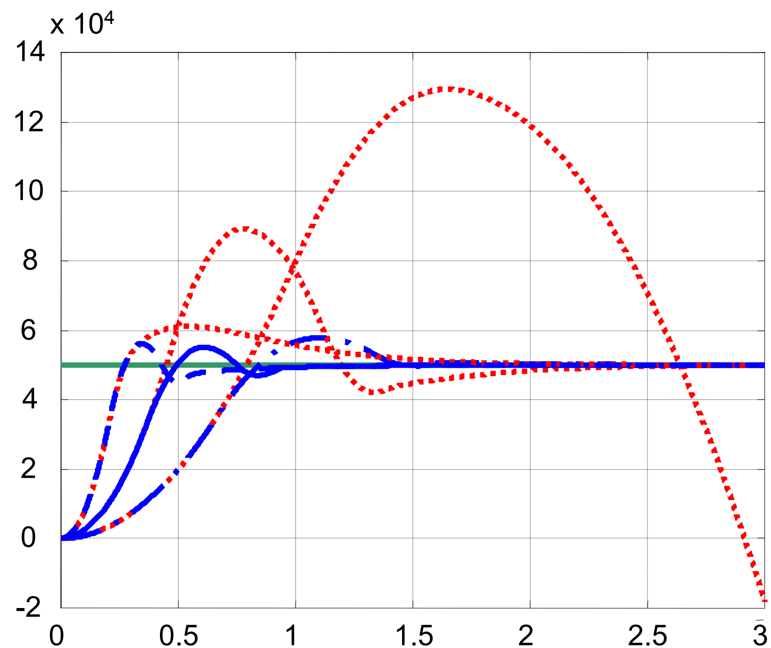
Using $N = 5$ pole-zero cells to approximate the fractional part of K_y

$$K_y(s) = 1e^5 \frac{1.24\,s^7 + 1.62\,e^3 s^6 + 4.53\,e^5 s^5 + 3.75\,e^7 s^4 + 1.18\,e^9 s^3 + 1.47\,e^{10} s^2 + 6.64\,e^{10} s + 9.68\,e^{10}}{s^8 + 6.53\,e^3 s^7 + 1.31\,e^7 s^6 + 8.01\,e^9 s^5 + 1.34\,e^{12} s^4 + 6.30\,e^{13} s^3 + 9.59\,e^{14} s^2 + 5.68\,e^{15} s + 1.16\,e^{16}}$$

$$K_u(s) = \frac{19.05s - 30}{s^2 + 10.95s + 30}$$



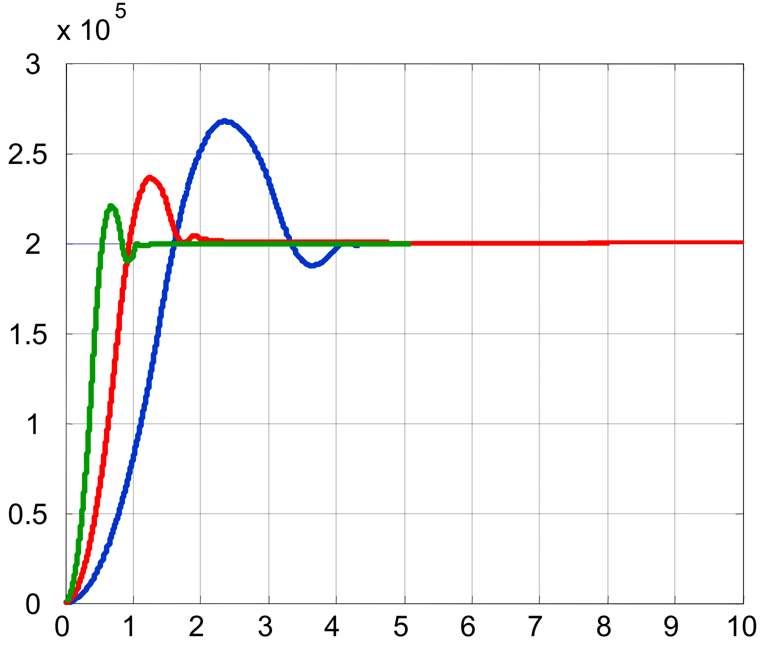
➔ The new nonlinear fractional controller is as robust for great signal responses as the linear fractional controller is for small signal responses.



Step responses obtained with and without the windup compensation system

Illustrative example (4/4)

Step responses obtained for ω_{D1} = 30rad/s when the reference signal varies from 0 to 2.10^5



The overshoot varies from 12 to 34%

➔ To limit the overshoot, the value of the tuning parameter ω_{D1} has to be adapted to the greatest value of the reference signal variation.

Conclusion

- It's very easy to include a windup compensation system in a 1st generation CRONE control-system

- The robustness of the linear control-system is preserved

- The non-linear control-system is robust

- The complexity of the modified 1st generation CRONE control-system is close to that of the linear 1st generation CRONE control-system

- The same frequency domain methodology can be applied to modify 2nd or 3rd generation control-systems obtained from the difference between an optimal open-loop frequency response and a nominal plant frequency response