LABORATOIRE ENERGETIQUE ET PHENOMENES DE TRANSFERT
LABORATOIRE INTERETABLISSEMENTS TREFLE

Phase reconstruction in non destructive thermal characterization

Andrzej Kusiak
Jean Luc Battaglia
Jean-Christophe Batsale
Jean-Jacques Serra

" Fractional systems and their applications "

ENIT - Tarbes - march 25th & 26th, 2004

General procedure

- Thermal Non Destructive Testing
- Thermophysical properties estimation

→ Inverse methods

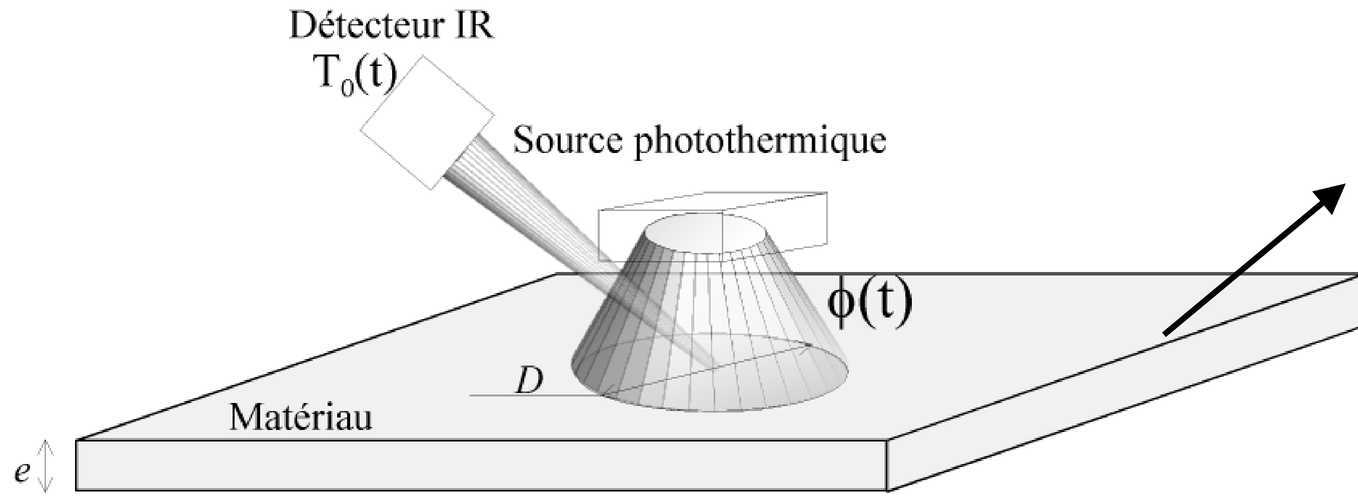
Adapted experiment realization
(Front face measurements)

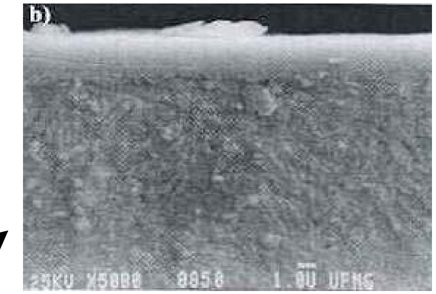
Signal / Image Processing
(Model)

- Work hypothesis called ‘comfortable’ : Linear and stationary system
- Multi scale systems characterization: Time scale concerns several decades

Experimental

Front face characterization





Experiment leads to the transfer model in the form: $\bar{T}_0(s) = H_\gamma(s)\bar{\varphi}(s)$

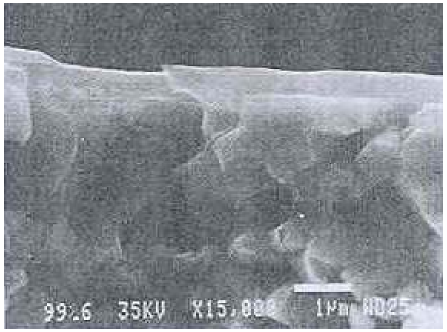
in the Laplace domain.

Transfer function: $H_\gamma(s)$ includes the thermophysical properties

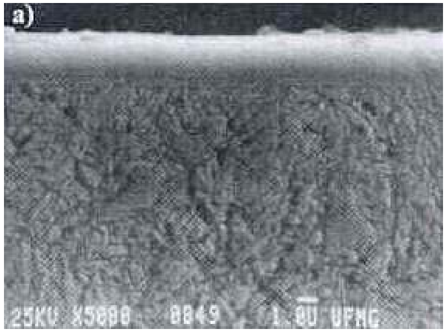
$$\gamma = \{\lambda, \alpha, R_c, e...\}$$

Example: Micrometric coatings characterization

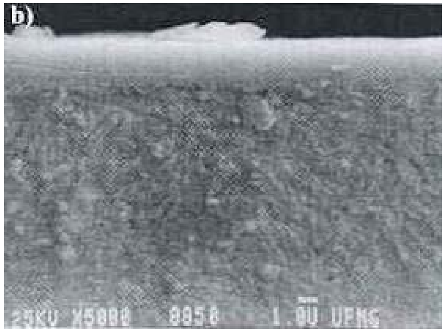
- PVD Coatings
- CVD Coatings



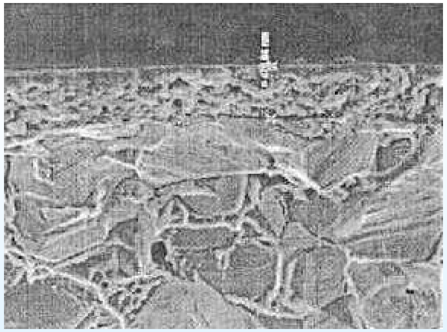
TiN 0,5~2 μm



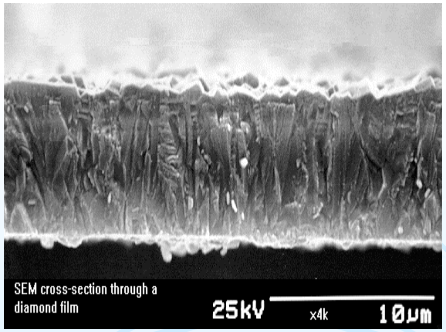
TiAlN ~2 μm



CrN ~2 μm

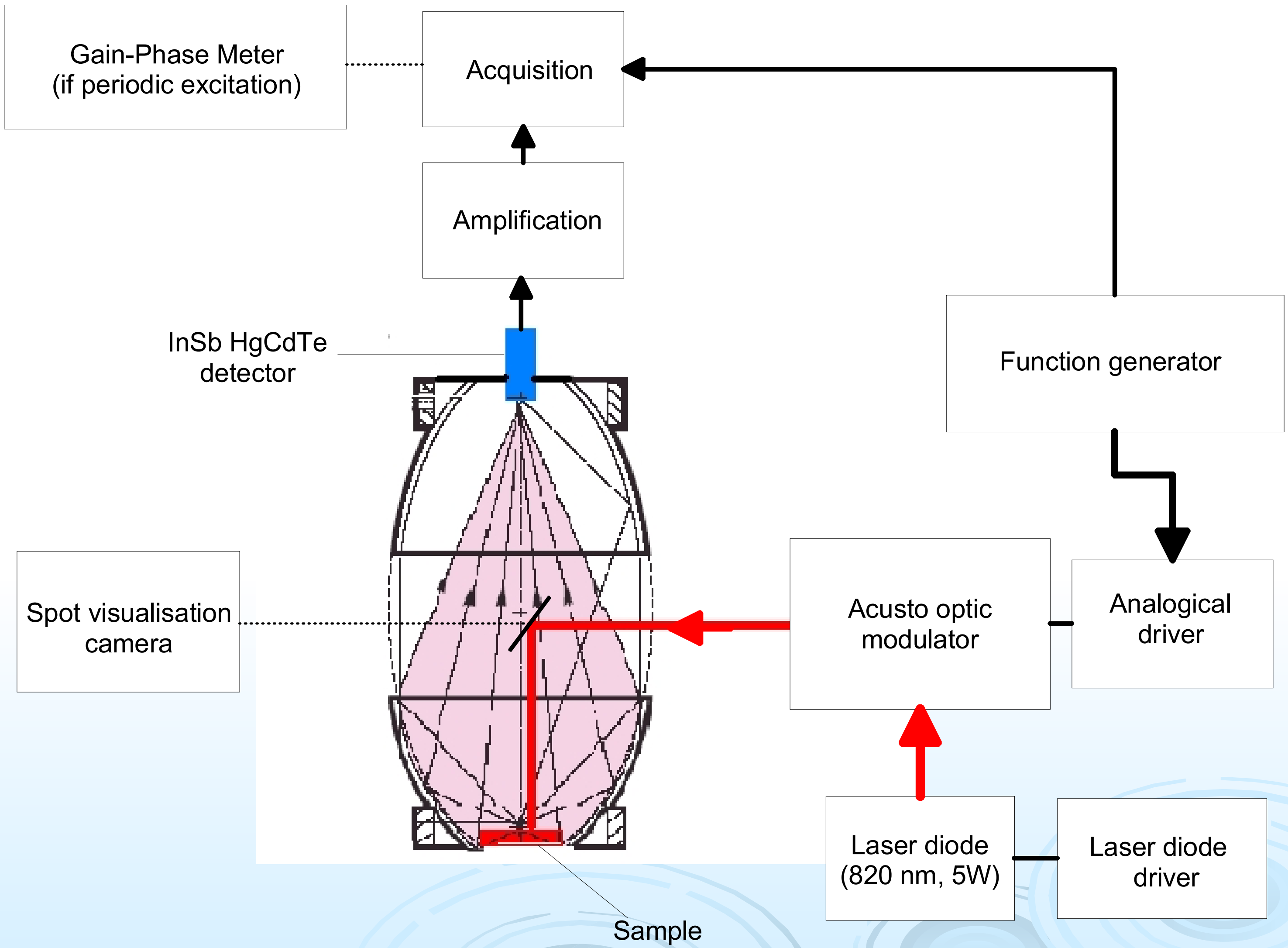


Diamond 5~20 μm

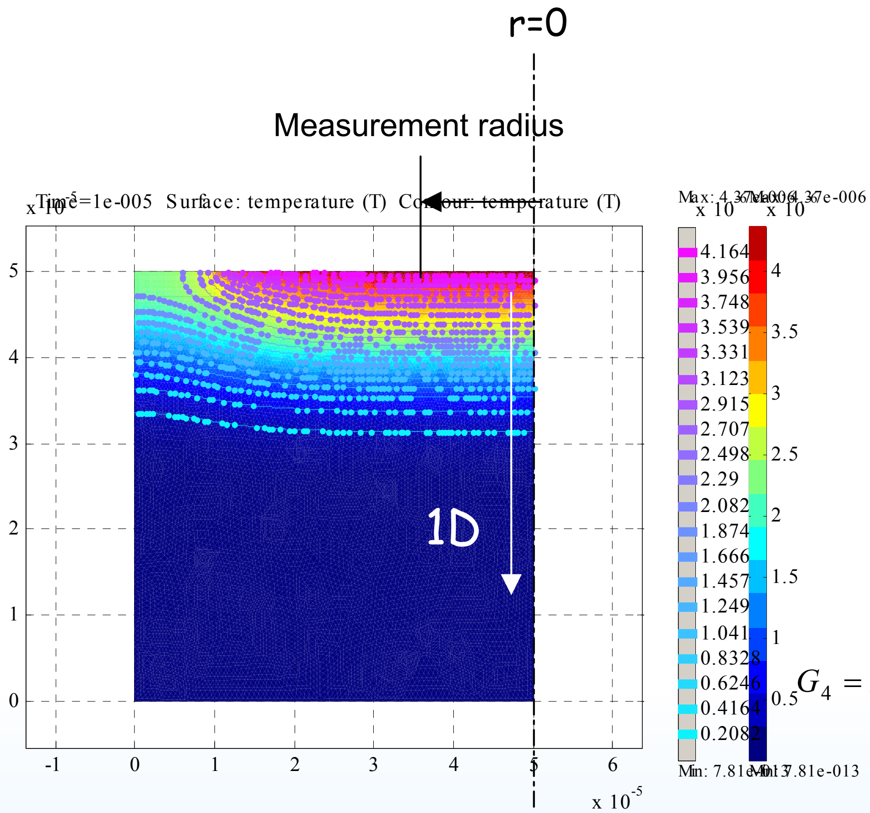


Adhesion deposit-substrate, deposit thickness???

Experimental setup developed in TREFLE (ex LEPT)



Reference transfer function



Analytical expression of the transfer function

$$\overline{T_0}(s) = H_\gamma(s)\overline{\varphi}(s)$$

with

$$H_\gamma(s) = \frac{G_0 + G_1 + G_2}{G_3 + G_4 + G_5}$$

$$G_0 = \cosh(k_s e_s)\cosh(k_d e_d) + R_c \lambda_s k_s \sinh(k_s e_s)\cosh(k_d e_d)$$

$$G_1 = \frac{\lambda_s k_s}{\lambda_d k_d}\sinh(k_s e_s)\sinh(k_d e_d)$$

$$G_4 = R_c \lambda_s k_s \sinh(k_s e_s)\lambda_d k_d \sinh(k_d e_d) + \lambda_s k_s \sinh(k_s e_s)\cosh(k_d e_d)$$

$$G_3 = \lambda_d k_d \cosh(k_s e_s)\sinh(k_d e_d)$$

$$G_2 = h\left(\frac{1}{\lambda_s k_s}\sinh(k_s e_s)\cosh(k_d e_d) + R_c \cosh(k_d e_d)\cosh(k_s e_s) + \frac{1}{\lambda_d k_d}\sinh(k_d e_d)\cosh(k_s e_s)\right)$$

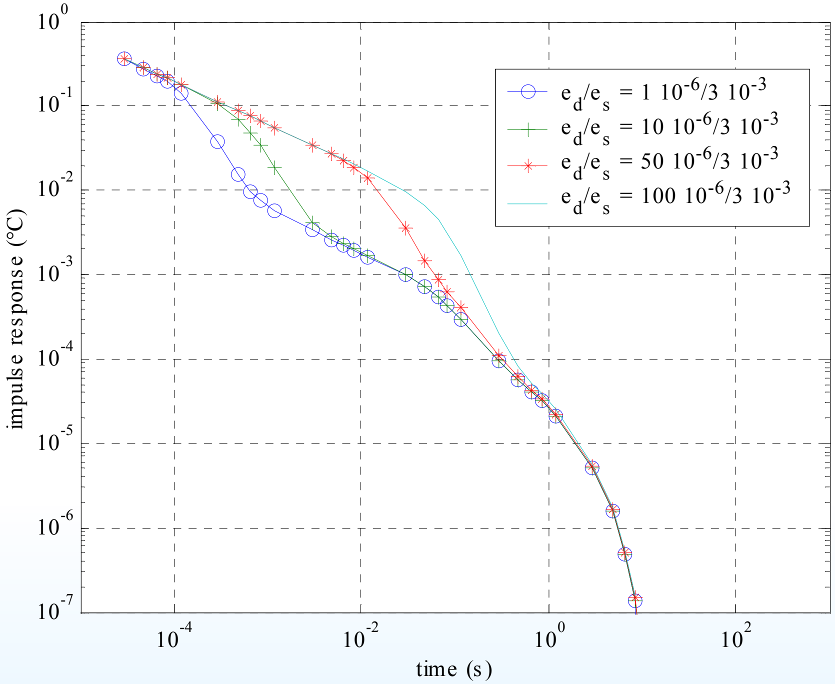
$$k_d = \sqrt{\frac{s}{\alpha_d}}$$

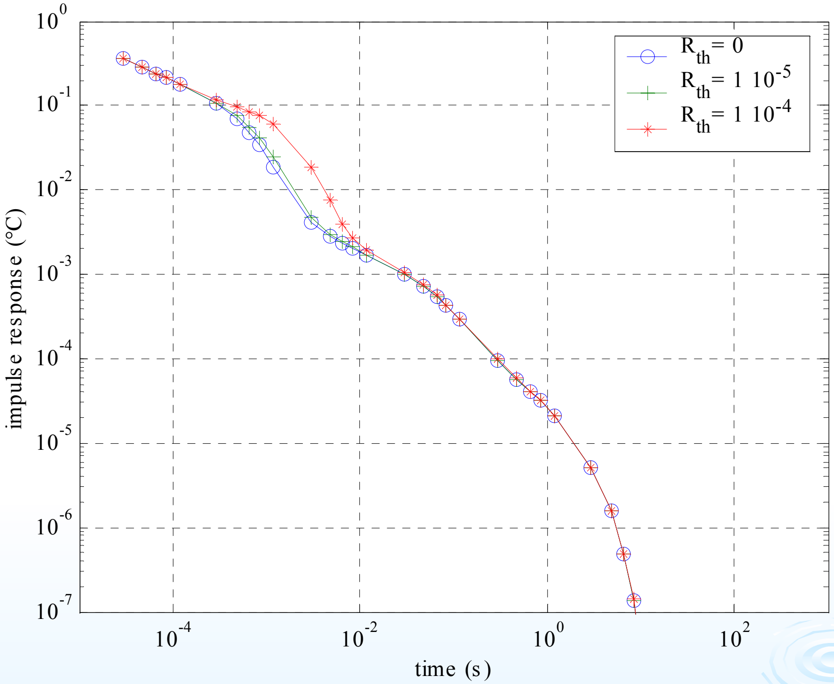
$$G_5 = h\left(\frac{\lambda_d k_d}{\lambda_s k_s}\sinh(k_d e_d)\sinh(k_s e_s) + R_c \lambda_d k_d \sinh(k_d e_d)\cosh(k_s e_s) + \cosh(k_d e_d)\cosh(k_s e_s)\right)$$

$$k_s = \sqrt{\frac{s}{\alpha_s}}$$

(adhesion) R_c, (thickness) e_d ???

Impulse response





(Impulse responses computed for different ratios e_d/e_s and different values of R)

In reality…

We have the luminance $L_0(t)$ instead the temperature and the current $i(t)$ (the IR detector measures a luminance and the laser is driven by current)

In theory $$\overline{T_0}(s) = H_\gamma(s)\overline{\varphi}(s)$$

In practice $$\overline{L_0}(s) = H_\gamma(s)\,\overline{i}(s)$$

Meanwhile, if one remains in the detectors linearity domain (small variation of the heat flux and the temperature), the following relations can be written:

$$\overline{T_0}(s) = a_\lambda \overline{L_0}(s) \quad \overline{\varphi}(s) = b\overline{i}(s)$$ where $\{a_\lambda, b\}$ are constants

Thanks to, the phase between $L_0(s)$ and $i(s)$ is the same for the temperature and the heat flux

From, the phase interests!!!

Two ways (classical) for the phase obtaining…

<u>Photothermal experiment</u>

Excitation:

Dirac

Impulse response *h(t)* obtained directly

then, to the frequency domain

$$H(j\omega) = \mathrm{FFT}\left[h(t)\right]$$

Periodic

Reconstruction of the transfer function $H(j\omega)$ (amplitude and phase)

However…

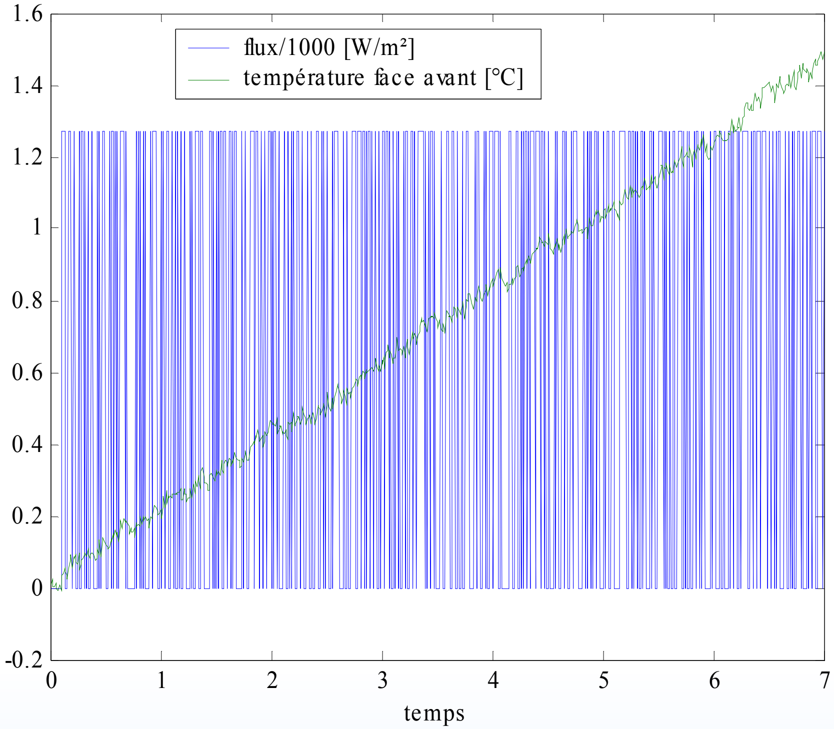
The linearity hypothesis difficult to respect

Long experiment

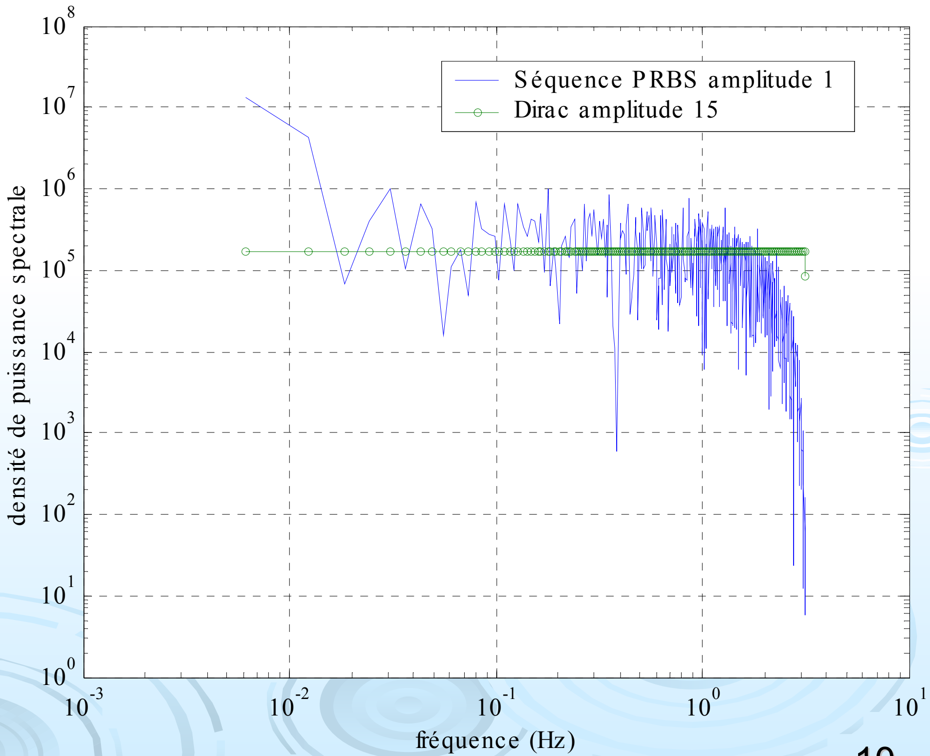
<u>and the phase reconstruction</u>

$$\mathrm{phase}(\omega) = \mathrm{Arg}\left[H(j\omega)\right]$$

How to do it easier and faster? By application of a random heat flux.



Pseudo-random binary sequence (PRBS)



Power spectral density

$$PSD(f) = \left|\bar{\phi}(j\omega)\right|^2$$

$$\text{with } \bar{\phi}(j\omega) = F\left[\phi(t)\right] = \int_{-\infty}^{\infty} \phi(t)\; e^{-j\omega t}\,dt$$

Now, the objective is to define a model adapted to the studied configuration

The classical PDE equations in the thermal field generally do not ensure (except simple configurations) a solution continue in time.

However, there is in 1D a solution obtained by application of the Laplace transform to the time variable

$$\overline{T}\left(x=0,s\right)=\mathrm{H}_{\gamma}\left(s\right)\overline{\varphi}\left(s\right)$$

$$\overline{T}\left(x=0,s\right)=\int_{0}^{\infty}T\left(x=0,t\right)\exp\left(-st\right)\mathrm{d}t$$

$$\overline{\varphi}\left(s\right)=\int_{0}^{\infty}\phi\left(t\right)\exp\left(-st\right)\mathrm{d}t$$

It is enough just to replace $s=j\omega$ in order to obtain easily the phase

For the configurations 2D and 3D, additionally the integral transforms (Fourier, Hankel) are applied on the space variables

Nevertheless, the parameters γ are often linearly dependent and the estimation becomes non linear vis-à-vis these parameters

So, more sophisticated modeling is necessary

• Reliable (coherent with physical diffusion phenomena, in particular vis-à-vis the system asymptotic behavior)

• Leading to a fast parameters estimation (linear least squares)

The solution consists in adaptation a technique inspired from the system identification domain

The use of the transfer representation in the form of mathematical relation where all the parameters are linearly independent and their estimation is linear

Heat transfer representation using the non integer derivation operator

Thesis: O. Cois, L. Puigsegur

- Whatever boundary condition type, the temperature at any point of the system can be expressed as the heat flux function (for a second kind condition) in the form of relation between the non integer derivatives of 0,5 order of these two quantities:

$$\sum_{i=M_0}^{\infty} \alpha_i \, {}^{\mathrm{x}}\mathrm{D}^n T(\mathbf{x},t) = \sum_{i=L_0}^{\infty} \beta_i \, {}^{\mathrm{x}}\mathrm{D}^n \phi(t)$$

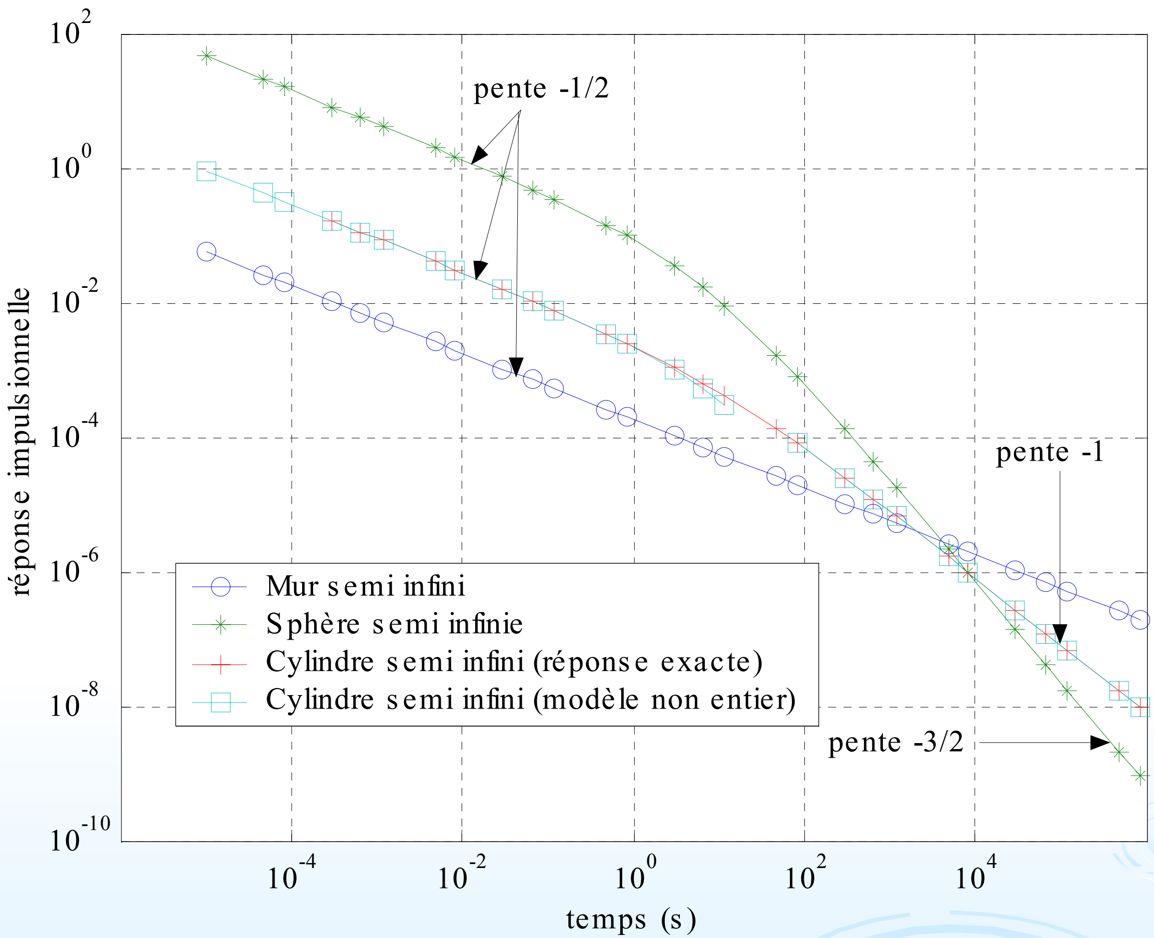
with

$$n = \frac{i}{2}$$

This model is a continue and exact representation of the heat diffusion process

Small times behavior

$$T_0(t) = \beta_0 \, \mathrm{I}^{0,5} \, \phi(t)$$



The modeling in the non integer operator sense is associated to the “semi infinite equivalent medium” (small times for a finite media)

The non integer model parameters are expressed explicitly as the functions of thermophysical parameters of the system.

Parameter	Expression of the parameter as a function of thermo physical proprieties	Parameter	Expression of the parameter as a function of thermo physical proprieties
β_0	$5\alpha_d^{3/2}\alpha_s^{3/2}\lambda_d$	α_2	$4\alpha_s^{1/2}\lambda_d(\alpha_s e_d\lambda_d + \alpha_d^{3/2}e_s\lambda_s)$
β_1	$4\lambda_d\left(e_d\alpha_d\,\alpha_s^{3/2} + e_s\alpha_d^{3/2}\alpha_s\right)$	α_3	$4\alpha_d^{1/2}\alpha_s^{1/2}e_d e_s\lambda_d(\alpha_d^{1/2} + \alpha_s^{1/2})$
β_2	$2e_s\alpha_d\alpha_s^{1/2}(2e_d\alpha_s^{1/2}\lambda_d + R_c\alpha_d^{1/2}\lambda_d\lambda_s + 2\alpha_d^{1/2}e_d\lambda_s)$	α_4	$4R_c\alpha_d^{1/2}\alpha_s^{1/2}e_s\lambda_d^2\lambda_s$

⋮ ⋮

Attention: the thermo physical parameters are interdependent in expression of the non integer model parameters

The number of parameters can be significantly reduced according to the required precision of the accordance between the asymptotic behaviors

$$H_0(s) = \frac{\sum_{i=0}^{\infty} \beta_i \; s^{i/2}}{\sum_{i=2}^{\infty} \alpha_i \; s^{(i+1)/2}}$$

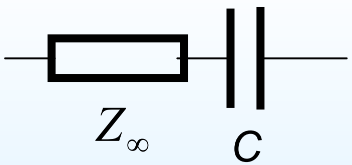
Complete fractional development

Reduction

$$H_{0,1} = \frac{\beta_0 + \beta_1 s^{1/2}}{\alpha_2 s}$$

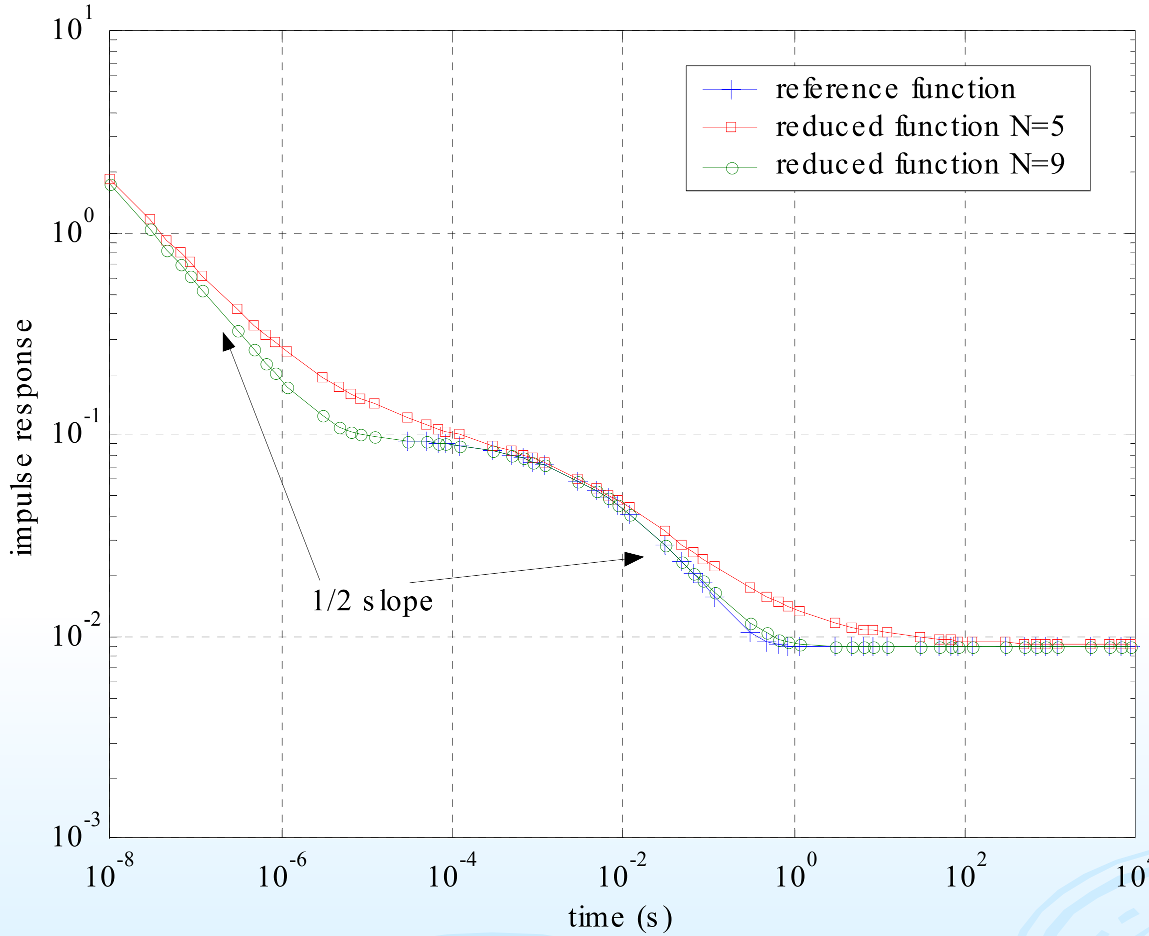
The reduced model describes the systems asymptotic behaviors and ensures their accordance

Accordance



$$H_{0,2} = \frac{\beta_0 + \beta_1 s^{1/2} + \beta_2 s}{\alpha_2 s + \alpha_3 s^{3/2}} \longrightarrow \left[\alpha_2 \; \mathrm{D} + \alpha_3 \; \mathrm{D}^{3/2}\right] T(0,t) = \left[\beta_0 \; + \beta_1 \; \mathrm{D}^{1/2} + \beta_2 \; \mathrm{D}\right] \phi(t)$$

Impulse responses for a deposit-substrate system



The parameters are linearly independent and can be identified in the linear least square sense (the derivation order is known)

- Front face temperature measurement

$$Y_0(t) = T_0(t) + e(t)$$

- Expression of $Y_0\ (t)$ in linear regression form

$$\mathrm{D}^{M_0/2} Y_0(t) = \mathbf{H}(t)\ \boldsymbol{\theta} + \varepsilon(t)$$

With:

- Linear regression matrix include successive derivatives of the temperature and the heat flux

$$\mathbf{H}(t) = \left[-\mathrm{D}^{(M_0+1)/2} Y_0(t) \quad \dots \quad -\mathrm{D}^{M/2} Y_0(t) \quad \mathrm{D}^{L_0/2}\phi(t) \quad \mathrm{D}^{(L_0+1)/2}\phi(t) \quad \dots \quad \mathrm{D}^{L/2}\phi(t) \right]$$

- Vector of parameters

$$\boldsymbol{\theta} = \left[\alpha_{M_0+1} \quad \dots \quad \alpha_M \quad \beta_{L_0} \quad \dots \quad \beta_L \right]^T$$

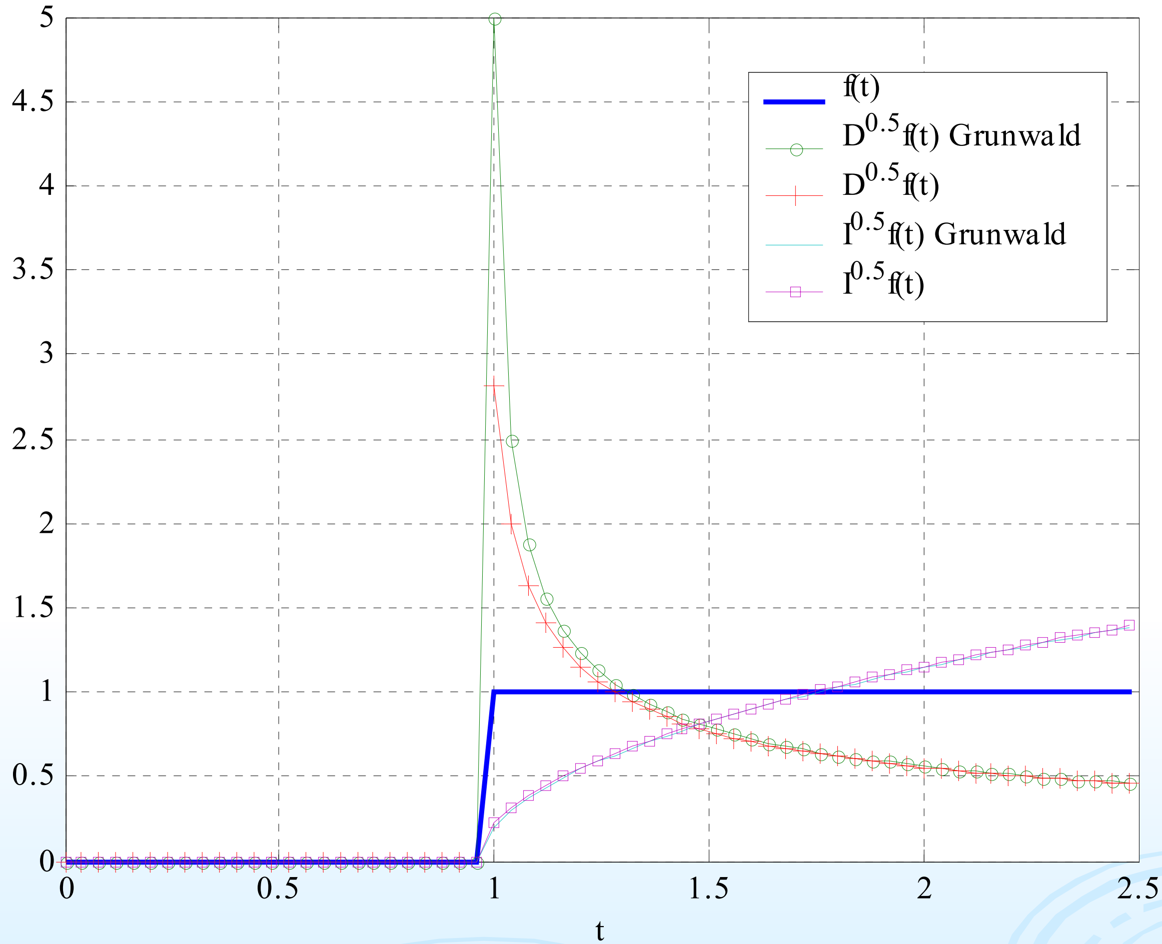
- Residue

$$\varepsilon(t) = \sum_{i=M_0}^{M} \alpha_i \quad \mathrm{D}^{i/2} e(t)$$

Identification in the least squares sense

$$\hat{\boldsymbol{\theta}} = \left(\mathbf{H}_K{}^T \mathbf{H}_K \right)^{-1} \mathbf{H}_K{}^T \mathbf{Y}_K$$

Problem : the derivation of the experimental data in the regression matrix amplifies significantly the measurement errors:



Sampling interval 0.04sec

It is necessary to filter the measured data: e. g. state filter.

New expression of the non integer model

$$H_0(s) = \frac{\sum_{i=L_0}^{L} \beta_i \quad s^{i/2}}{\sum_{i=M_0}^{M} \alpha_i \quad s^{i/2}}$$

Dividing the numerator and denominator of the transfer function by: $s^{M/2}$

One finds:

$$H_0(s) = \frac{\sum_{i=L_0}^{L} \beta_i \quad s^{-(M-i)/2}}{\sum_{i=M_0}^{M} \alpha_i \quad s^{-(M-i)/2}}$$

$$\boxed{\sum_{i=M_0}^{M} \alpha_i \quad \mathrm{I}^{(M-i)/2} T_0(t) = \sum_{i=L_0}^{L} \beta_i \quad \mathrm{I}^{(M-i)/2} \phi(t)} \quad \alpha_0 = 1$$

Thus, the new expression of regression matrix becomes:

$$\mathrm{H}'(t) = \left[-\mathrm{I}^{(M-M_0-1)/2} Y_0(t) \;\; \ldots \;\; Y_0(t) \quad \mathrm{I}^{(M-L_0)/2} \phi(t) \;\; \ldots \;\; \mathrm{I}^{(M-L)/2} \phi(t) \right]$$

From now, the data filtering is not necessary!

Another problem: huge quantity of the data for treatment.

The recursive least square method is used to estimate the unknown parameters (the Kalman filter) :

$$\hat{\boldsymbol{\theta}}(t) = \hat{\boldsymbol{\theta}}(t-1) + \mathbf{L}(t)\ \left[y(t) - \mathbf{H}'(t)\ \hat{\boldsymbol{\theta}}(t-1) \right]$$

The estimated parameters confidence domain:

$$\mathbf{cov}(\boldsymbol{\theta}) \approx \mathbf{P}(t)\frac{\sigma_y}{2}$$

Impulse response reconstruction

Non integer system identification from temperature Y_0(t) and heat flux ϕ(t) measurements from a random excitation

$$\sum_{i=M_0}^{M} \alpha_i \ \mathrm{D}^{i/2} T_0(t) = \sum_{i=L_0}^{L} \beta_i \ \mathrm{D}^{i/2} \phi(t)$$

Then the impulse response is calculated introducing ϕ(t)=Dirac(t)

$$\sum_{i=M_0}^{M} \alpha_i \ \mathrm{I}^{i/2} h(t) = \sum_{i=L_0}^{L} \beta_i \ \frac{t^{i/2-1}}{\Gamma(i/2)} \longrightarrow h(t)$$

The standard deviation on the identified parameters lead to express the error on the impulse response:

$$\Delta h = \sum_{i=L_0}^{L} \Delta\beta_i I^{i/2} \delta(t) - \sum_{i=M_0}^{M} \Delta\alpha_i I^{i/2} h(t) - \sum_{i=M_0}^{M} (\alpha_i + \Delta\alpha_i) I^{i/2} \Delta h(t), \quad \alpha_0 = 1, \ \Delta\alpha_0 = 0$$

(numerical resolution using Grünwald approximation)

Phase reconstruction

In the same manner

$$H_0(s) = \frac{\sum_{i=L_0}^{L} \beta_i \quad s^{i/2}}{\sum_{i=M_0}^{M} \alpha_i \quad s^{i/2}}$$

thus,

$$\theta_0(j\omega) = \underbrace{\frac{\sum_{i=L_0}^{L} \beta_i \quad (j\omega)^{i/2}}{\sum_{i=M_0}^{M} \alpha_i \quad (j\omega)^{i/2}}}_{H(j\omega)} \psi(j\omega)$$

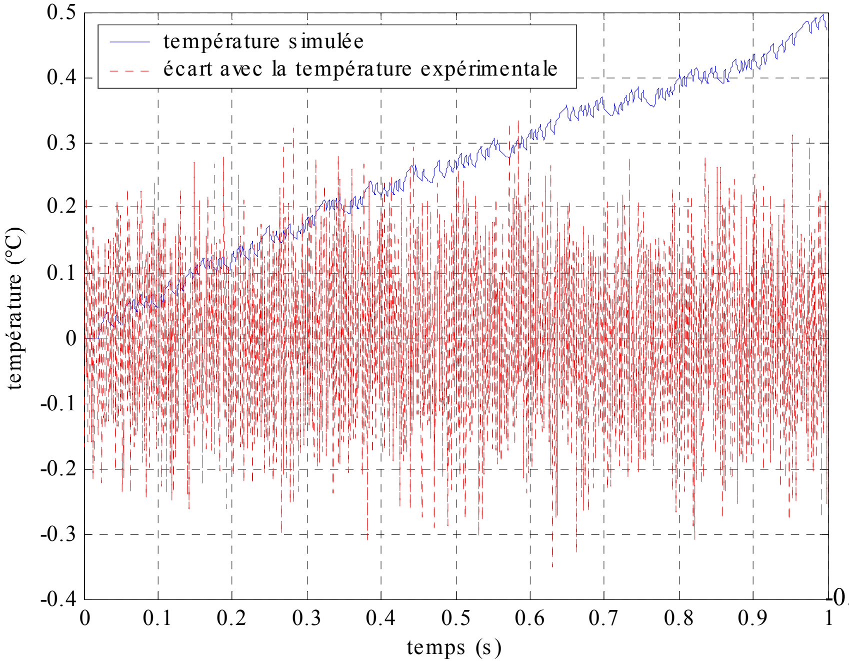
Then, the phase can be reconstructed as:

$$\varphi(\omega) = \arg\left[H(j\omega)\right]$$

The standard deviation on the identified parameters leads to the error on the phase reconstruction

(numerical resolution using Grünwald approximation)

Application

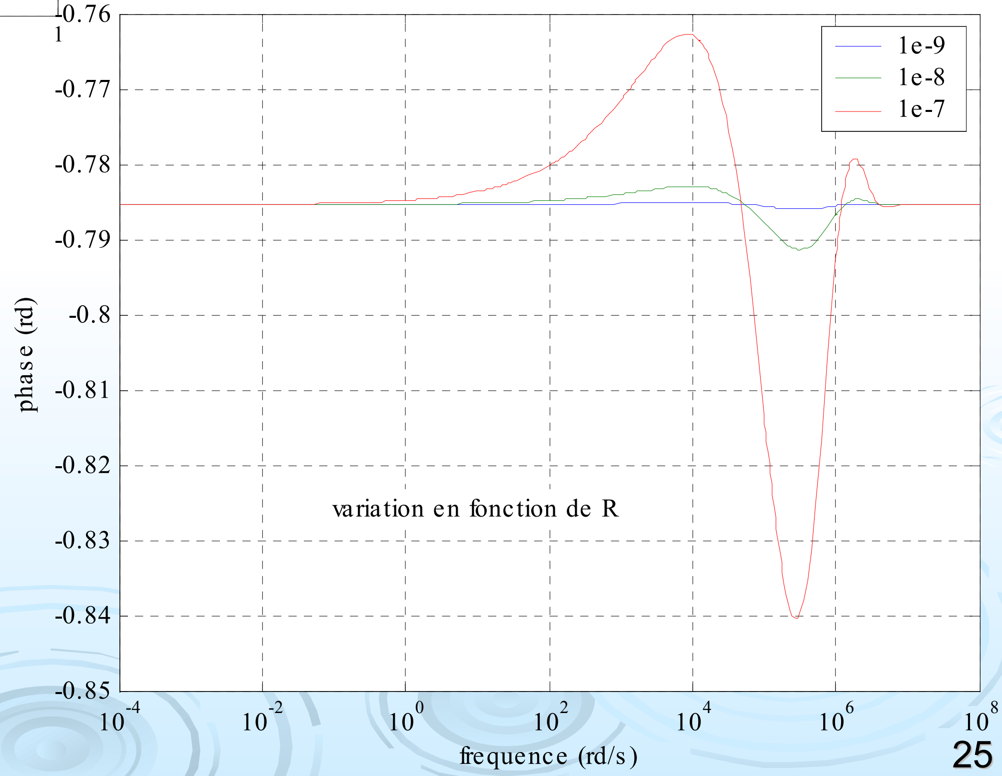


Experiment

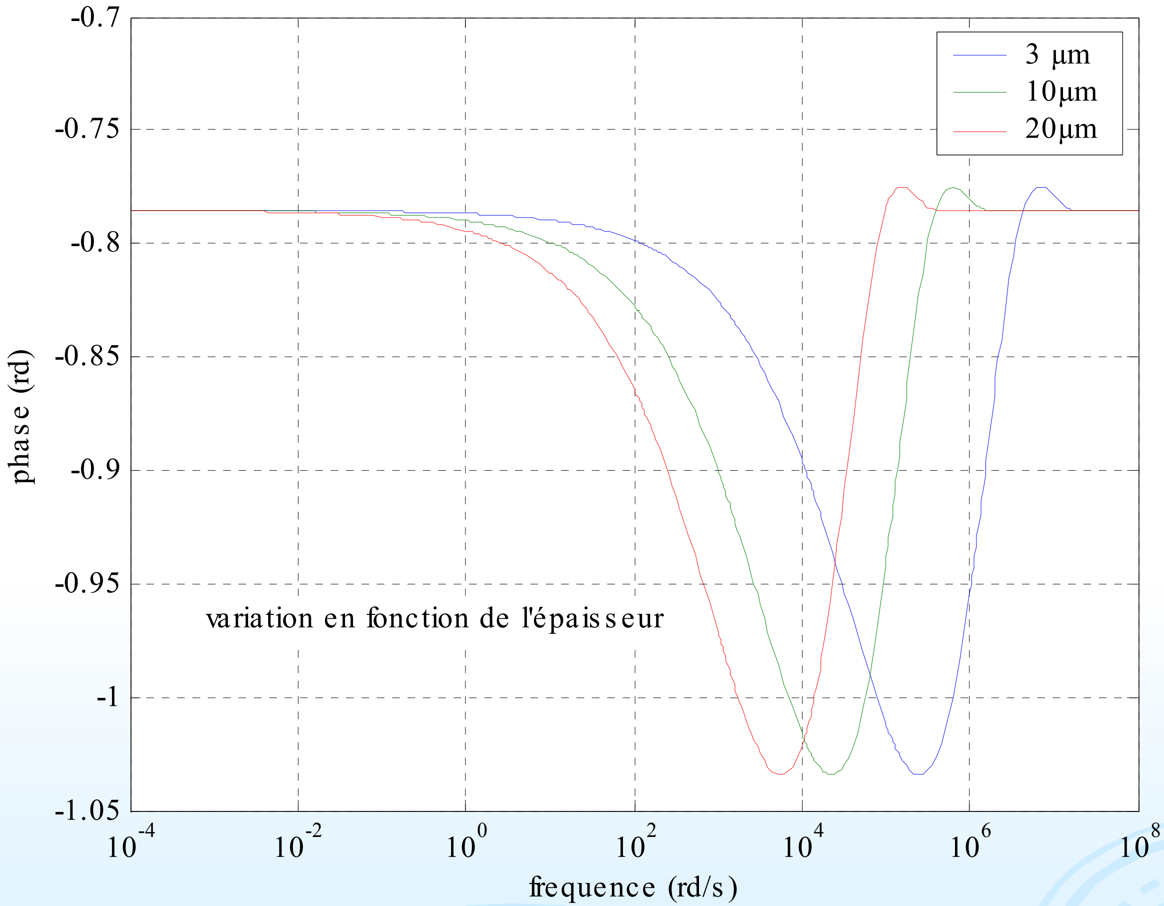
Parameters identification

$$H(j\omega)=\frac{\sum_{i=L_0}^{L}\beta_i\ (j\omega)^{i/2}}{\sum_{i=M_0}^{M}\alpha_i\ (j\omega)^{i/2}}$$

Phase reconstruction using the identified model



Phase variation according to the deposit thickness



Then, in order to find R_c and e_d

Identified model

$$\theta_0(j\omega) = H(\gamma_n, j\omega)\ \psi(j\omega)$$

Analytical 'exact' model

$$\zeta_{\alpha_d,\,R_c} = \mathrm{Arg}\left[H(\gamma_n, j\omega)\right]$$

The phase analytical expression

Thermal contact resistance estimation and the deposit thickness by comparison with the reconstructed phase

$$\hat{R}_c = \min\left\|\zeta_{\alpha_d,R_c}{}^2 - \tilde{\zeta}^2\right\|$$

The Levenberg-Marquardt non linear minimization algorithm

The use of simple relations $R_c=f(\alpha_i\,,\,\beta_i)$ and $e_d=f(\alpha_i,\,\beta_i)$

R_c and e_d

Conclusions

- The thermal characterization respecting the supposed linearity hypothesis
- Rapid experimental and the data treatment
- Small number of parameters in the reduced model
- A compact model can be obtained by simultaneous parameters and order estimation – this leads to non-linear estimation

Prospects