NON-INTEGER MOTION CONTROL:
APPLICATION TO AN XY CUTTING TABLE

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Abstract: A new approach to path tracking using non integer order differentiation applied to non-time-varying plants is proposed with its application on a XY cutting table. It permits the generation of optimal movement reference-input leading to a minimum path completion time, taking into account both the physical constraints of the actuators (maximum velocity, acceleration and torque) and the bandwidth of the closed-loop system. This reference-input results from the step response of a non integer order filter whose frequency response is that of an implicit generalized derivative filter. This new approach is complementary to CRONE control which synthesizes a robust control law, and which is based on real or complex non-integer order differentiation. The reference-input obtained is compared with spline function. Both are applied to an XY cutting table and actuator outputs compared. *Copyright © 2000 IFAC*

Keywords: robotics, control of mechatronic systems, motion control, advanced manufacturing technology, filtering techniques, movement generation, path tracking, minimum-time control, optimal control, non-integer order differentiation, XY cutting table.

1. INTRODUCTION

In plant control design (figure 1), there are two stages: *synthesis of the main control* to achieve the desired results in regulation, given in particular the input sensitivity of the plant; and *path planning* to generate control loop reference-inputs, given the constraints imposed by the task to be carried out by the plant and its physical limitations. The latter stage has two subdivisions: *path description* which defines geometrically the trajectory to be followed depending on the nature and environment of the task to be completed; *movement* or *input generation* which determines how the path is to be followed, given the desired results (e.g. path completion time) and the physical constraints of the actuators (velocity, acceleration, and maximum current and voltage). It is necessary then to determine an algorithm, which can calculate reference-inputs for the control loop while minimizing a given criterion. The criterion most often adopted, is path time.

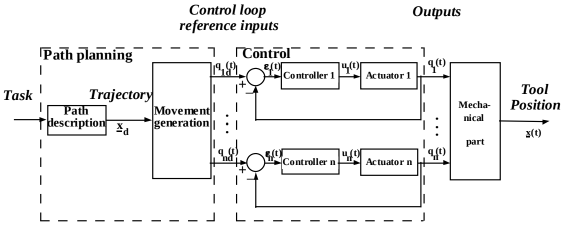

Fig. 1. Plant control design

Step type input references must be proscribed. This is because: on the one hand, the high frequencies of the power spectral density (PSD) are not given in the output, due to the low-pass character of the control loops, thus reducing the quality of tracking; and on the other hand, because these high frequencies solicit unacceptably high velocities and accelerations from the actuators and other tools.

Much work has been carried out on input generation. The polynomial approach (Dombre and Khalil, 1988; Khalil and Dombre, 1999) respects the limitations imposed by the maximal velocity and acceleration, but permit neither maximal values of speed and acceleration, nor minimal path completion time to be reached.

The Bang Bang approach (Dombre and Khalil, 1988; Khalil and Dombre, 1999) takes into account the same physical constraints but does provide a minimal path completion time. However, as for the polynomial approach, the dynamics (bandwidth) of the control loop are not taken into account, so overshoots can appear on the end actuator.

Although popular, these approaches are now being abandoned by some manufacturers as they not able to limit overshoots appearing for small displacements. A simple digital filter is often preferred, as it is easy to implement and to adapt for killing overshoots. It reduces the high frequency energy of the path planning signal using a low-pass filter with trial-and-error determined parameters. This type of path tracking, based on position step filtering, does not permit separate control over maximal values of velocity and acceleration, which stay proportional to the amplitude of the step applied.

When the control loop is perfectly defined, algorithms of Shin and Mac Kay (1985, 1987) or Bobrow, *et al.* (1985), allow the synthesis of the optimal actuator control that takes into account constraints on the inputs and the details of the dynamics manipulator. The dynamic model of the process must be designed by applying Lagrange formalism. The use of curvilinear abscissa allows reduction of the number of variables without loss of information. The minimal path time is determined from the phase curve using the Pontryagyn maximum principle. However, this is fastidious and must be done for each trajectory. On the other hand, this method provides no connection to tracking accuracy, except for some particular case (Kieffer, *et al.*, 1997). These disadvantages limit the development of these algorithms in industry.

When k points on it define the trajectory, the form of the curve between each point has to be determined, while respecting nominal speed and acceleration. A polynomial of interpolation can be calculated, but the degree of this polynomial increases with the number of points, and can be unmanageable. Cubic spline functions (order 3 piecewise polynomials) overcome this difficulty. They are now widely used in robotics (Kochan, 1998). They are minimal curvature curves (Dombre and Khalil, 1988) and the optimization proposed by Lin, *et al.* (1983), or De Luca, *et al.* (1991), based on the non-linear simplex optimization algorithm (Nedler and Mead, 1965) offers a complete-path reference solution. However, as in the polynomial or Bang Bang approach, the dynamics of the control loop are not taken into account: overshoots on the end actuator appear for small displacements.

A new approach to path planning using non-integer order differentiation (Oustaloup, 1983, 1995) applied to non-time-varying plants is developed by Melchior, *et al.* (1998, 1999). It permits the generation of optimal movement reference-input leading to a minimum path completion time, taking into account both the physical constraints of the actuators (maximum velocity, acceleration and torque) and the bandwidth of the closed-loop system. This reference-input results from the step response of a non integer order filter whose frequency response is that of an implicit generalized derivative filter. It permits the definition of analytical expressions of position, velocity and acceleration profiles and their maxima, from only two parameters: the non-integer order and the corner frequency of the filter. It can be implemented as a classical digital filter. It is synthesized in the frequency domain, thus the PSD of the position permits absolute control of the high frequency energy. This approach is complementary to CRONE control (Melchior, *et al*, 1996) which synthesizes a robust control law, and which is based on real or complex non-integer order differentiation.

Motion control using an implicit generalized derivative (IGD) position filter is easy to implement, as it is simply a numerical filter, and it efficiently kills overshoots, especially for small displacement (Melchior, *et al.*, 1998). To separate speed and acceleration control, an IGD speed filter has been developed allowing intermediate speed control for path tracking (Melchior, *et al.*, 1998, 1999). As the spline function, made up of one jerk step per point, is a reference in robotics, we decided to take and adapt its optimisation algorithms to develop an IGD jerk filter.

The remainder of this paper is divided into four sections. Section 2, gives the principle of the approach to motion using IGD jerk filters. The model and identification of the XY cutting table are presented in section 3. The application of the approach to the XY table is given in section 4. Simulation performance obtained using the IGD jerk filter and the spline function method are compared. The final section discusses the significance of the results.

2. IMPLICIT GENERALIZED DERIVATIVE (IGD) FILTER

2.1. IGD position filter

Polynomial interpolation and Bang Bang laws have a bandwidth that varies with the length of the displacement. Overshoots observed for small displacements are due to these variations. Numerical filters have a fixed bandwidth, allowing optimization in the frequency domain once and for all, and for all displacements, to limit end actuator vibration.

The IGD position filter for motion control generates a reference-input from the step reply of a low bandwidth filter described by the transmittance:

$$F(s) = \frac{1}{(1+\tau s)^n} \quad . \tag{1}$$

The use of real poles prevents frequency resonance.

The choice of identical poles allows the greatest possible energy on a given bandwidth (figure 2).

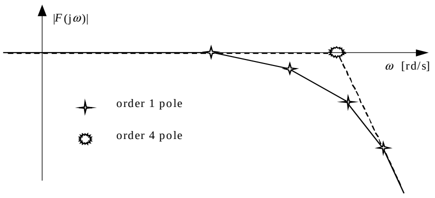

Fig. 2. Pole assignment for a maximum energy in a given pass-band

The filter given by expression (1), where parameter n is real and no longer restricted to being an integer, is an IGD filter (Oustaloup, 1983; Melchior *et al.*, 1996). Thus the filter given by (1) reduces energy of the signal at high frequencies by defining a bandwidth (time constant τ) and, through the continuous nature of the selectivity (real order n) as can be seen in figure 3. The optimization of parameters n and τ considers the static constraints (V_{nom}, A_{nom}, J_{nom}) and the dynamic constraints ($\omega_c<\omega_r$), to reduce resonance.

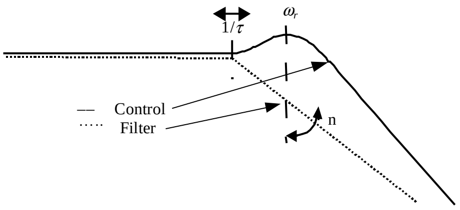

Fig. 3. Power spectral density assignment of the implicit generalized derivative filter compared to resonance frequency placement of the control loop to which it is applied

The IGD position filter methodology defines analytic profile expressions of position, speed, acceleration and their maxima, using only two parameters (n and τ); it ensures a continuous acceleration for an implicit order $n > 1$; it permits continuous optimization on the two parameters to limit resonance and bandwidth and generates an optimal reference-input that ensures minimal path time. It takes into account the static and dynamic physical constraints of the actuator and control loop to which reference-inputs are applied. This filter can be used to build either analog or digital filters. However, maximal values of speed, and acceleration are reached only punctually, so generation is under-optimal; maximal values of speed and acceleration reached are linked as they are both proportional to the amplitude of the step applied; only null intermediate speeds are possible for a reference-input sequence.

2.2. IGD Speed Filter

This filter is based on an IGD position filter. To uncouple acceleration from speed, and to obtain a constant nominal speed, an integration operator is added to expression:

$$\mathrm{F}(s)=\frac{1}{s(1+\tau s)^n} \quad . \qquad (2)$$

The principle of this method is given in figure 4.

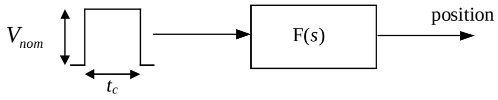

Fig. 4. Principle of motion control by implicit generalized derivative speed filter

The new transmittance, given by expression (2), uses a speed reference-input. The constant speed level is obtained from one crenel. The amplitude of the crenel is the constraint on the nominal speed. The duration t_c of this order is imposed by the length of the displacement:

$$t_c=\frac{L}{V_{nom}} \quad . \qquad (3)$$

It is thus possible to determine analytically n and τ which ensure that the reference-input generated respects nominal acceleration whatever the length of the displacement, while ensuring reduction of the resonance of the control loop. The minimization of the path time uses two criteria: response time, and the gap integral between the crenel and the reference-input.

The method has been extended for non-null initial and final speeds, to follow trajectories defined by k points (Melchior and Orsoni, 1999). The research of the optimal solution admits a linear formulation of the criterion and constraints for one dimension trajectories. Intermediate speeds are optimized, taking the whole path into account, by the G.B.Dantzig simplex method (non-linear algorithm or the *constr* Matlab© function in multi-dimension). The comparisons with spline function (Melchior and Orsoni, 1999; Melchior *et al.*, 2000) show that, using the non-integer approach, the inclusion in the optimization criteria of a dynamic constraint (closed-loop system bandwidth) permits not only better tracking, but also reduction of overshoots for small displacements

The IGD speed filter has the advantages of the IGD position filter: ease of implementation and use; efficient reduction of overshoots and saturation.

It is improved as: intermediate speed is controlled; nominal speed is held constant; the same filter is kept all along the trajectory.

2.3. IGD Jerk Filter

A direct application of Lin's algorithm (Lin *et al.* 1983) on the optimization of cubic spline functions is presented by Kochan (1998) as the reference method for motion control.

The spline function, made up of one jerk step per point, is a reference in robotics. The Bang Bang law, on jerk, classically defined for two points, can be considered as extended to n points.

The optimization algorithm is a non-linear simplex one. It determines minimal time intervals between each point, by taking into account nominal values of speed, acceleration and jerk. Increasing V_{nom}, A_{nom} and J_{nom} respectively by a factor λ, λ^2 et λ^3, the final time of the optimal solution is reduced by this same factor λ.

Given: $$\mathscr{L}\{\, f(\lambda t)\mathrm{u}(t)\,\} = \frac{1}{\lambda} F\left(\frac{s}{\lambda}\right) \quad , \qquad (4)$$

The more static the magnitude V_{nom}, A_{nom} et J_{nom}, the shorter the final time, but the greater the actuator bandwidth must be.

Static magnitude, V_{nom}, A_{nom} and J_{nom} , has no direct link with the bandwidth of the system: it is possible to find two actuators with the same static characteristics but with very different bandwidths. However, as indicated by property (4), the static magnitudes fix the reference-input bandwidth for spline functions. This bandwidth is not necessarily inferior to the actuator bandwidth.

This corroborates empirical verifications and simulations: cubic spline functions, as polynomial methods, do not reduce overshoots for small displacements. It can be noted that the nominal value of jerk can serve arbitrarily and empirically to limit the bandwidth of the reference-input, and is thus misused.

The spline function optimization algorithm is taken and adapted to produce the IGD jerk filter described in figure 5.

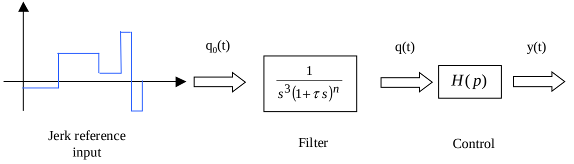

Fig. 5. Principle of motion control by implicit generalized derivative jerk filter

The dynamics of the actuator are taken into account by introducing a specific supplementary constraint: relaxation time. It is the duration required for the system to rejoin its asymptote (for a given precision). Each jerk step is controlled independently, as the reference-input bandwidth is included in the control loop bandwidth.

3. MODELISATION AND IDENTIFICATION OF AN XY CUTTING TABLE

3.1. *Description of the plant*

The study plant is a cutting table for leather or fabric, using laser or high-pressure water as cutting tools. Several types of tables exist, H-shaped, T-shaped, cross-shaped or XY tables, each with a specific field of application. The table considered here is an XY cutting table, whose operating principle is illustrated in figure 6.

This XY design determines the specific mechanical features of this type of table, which results in two study properties. First, the two degrees of freedom are independent. Second, the mechanical differences between the two degrees of freedom are not structural but parametric, in particular: carried mass: tool mass (Y) 2-20 Kg, beam plus tool mass (X) 30-60 Kg; stiffness of the driving belts; characteristics of the driving motors M_x and M_y .

Only the degree of freedom on the Y-axis is modeled, as the X-axis is deduced from it by simply changing the parameter values.

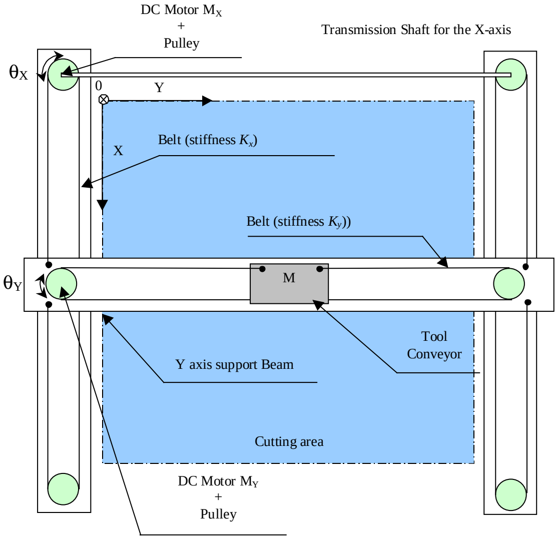

Fig. 6. Schematic representation of the XY cutting table

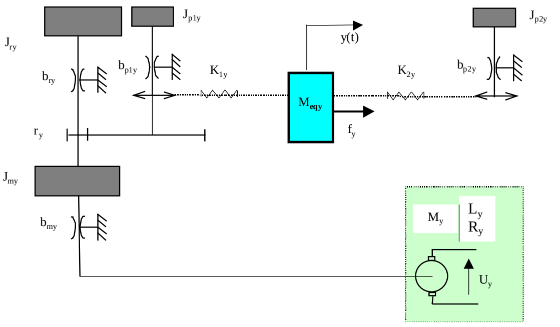

Fig. 7. Functional design of the Y axis

3.2. *Dynamic model design*

The dynamic model design of the Y axis is given in figure 7. As the belts are not perfectly rigid, the Y-axis system has two degrees of freedom θ_y(t)and y(t). By applying the Laplace transform on the mechanical equations obtained by the Lagrange equations of the system, we obtain:

$$\begin{aligned}&\left\{\left(r_y{}^2 J_{p1y} + J_{my} + J_{ry}\right)s^2 + \left(b_m + b_{ry} + r_y{}^2\, b_{p1y}\right)s \right. \\ &\left. + r_y{}^2 R_{py}{}^2 K_{eqy}\right\}\theta_{my}(s) = K_{eqy}\, r_y\, R_{py}\, Y(s) + C_m Y(s)\end{aligned} \qquad (5)$$

$$\begin{aligned}&\left[\left(M_{eqy} + \left(J_{p2y} / R_{py}{}^2\right)\right)s^2 + \left(b_{p2y} / R_{py}{}^2\right)s + K_{eqy}\right] Y(s) \\ &= K_{eqy}\, r_y\, R_{py}\, \theta_{my}(s) - f_y(s)\end{aligned} \qquad (6)$$

with $$U_y(s) = E_y(s) + (L_y\, s + R_y)\, I_y(s) \quad , \qquad (7)$$

$$E_y(s) = s\, K_{ey}\, \theta_{my}(s) \quad , \qquad (8)$$

$$C_{my}(s) = K_{cy}\, I_y(s) \quad . \qquad (9)$$

3.3. *Identification*

Dynamic identification determines numerical parameter values of the model, and their variation range according to functioning conditions. We are not permitted to disclose these values.

SANYO DC motors are used. The modeling of the belt stiffness is based on two assumptions:

- the belt is sufficiently pre-stressed for stretching not to create a sagging phenomenon;

- stiffness calculation only takes into account the elasticity of the steel cables forming the belt, as the polyurethane support adds negligible stiffness in parallel with the cable stiffness (combination law of two springs mounted in parallel). Hooke's law allows expression of the various stiffness constants versus the features of the belt considered, and to the straight positions of the tool conveyor. Hence, the stiffness of the belt along the Y degree of freedom:

$$K_{eqy}(Y) = K_y\left(\frac{1}{Y} + \frac{1}{2.L - Y}\right) \qquad (10)$$

where *Ky* is the constant characteristic of the cable material, called Young modulus.

To validate the model more easily, a point of functioning is defined, at the point of least variation of the stiffness. The identification of inertia in rotation, and masses in translation, is determined from the nature and dimension of each element. Dry friction values are measured on the cutting table, using a *peson*. Current consumption of the DC motors provides the viscous friction coefficients, which are proportional to axle speed, and displacement tests at constant speed along each axis can be carried out. However, the influence of viscous friction on the system is weak and difficult to measure. Evaluations have been defined by sameness with others processes.

3.4. Model validation

From the identified model of the plant, time and frequency responses of the model can be simulated, to compare with real readings from the table. A 100mm diameter circle of cotton dress material to be cut (in our test drawn on paper by a pencil, replacing the cutting head) is taken as a validation example.

The nominal value for acceleration is 10m/s^2 and the nominal speed value is 1m/s. The reference input, the controller signals, and the actuator outputs are digitized. The reference input is used as input for our simulation model, and the resulting simulated controller signals and actuator outputs compared to those of the real cutting table (figures 8–11).

These results show that the model is partially validated. The approximations used in the model, notably the viscous and dry friction values, are under-estimated. However, the model gives a good evaluation of the end-actuators and end-manipulators outputs.

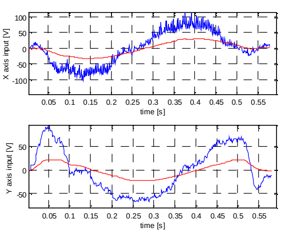

Fig. 8. Plant inputs for 0.1m circle (joint space)

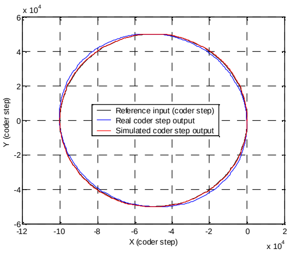

Fig. 9. Coder outputs for 0.1m circle

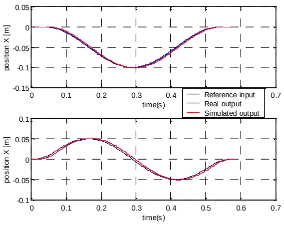

Fig. 10. Axis positions for 0.1m circle

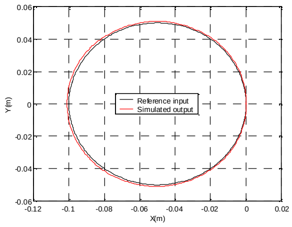

Fig. 11. End-manipulator output for 0.1m circle (operational space)

4. COMPARISON OF IGD JERK FILTER AND SPLINE FUNCTION

As can be seen in the Bode diagrams (figure 12), for the frequency responses of the X and Y control axes, the resonance values and the resonance pulsation values are nearly identical: 5dB and 110rad/s respectively. These values are parameters that will be used to define the reference input for the IGD jerk filter.

The outputs resulting from the IGD jerk filter and spline function reference inputs, on the model of the cutting table, are compared. Comparisons are made in the two dimensions of the operational plane. The comparison protocol is based on two classical figures (square and circle) of small and medium dimensions for the cutting table. Simulations are shown in the figures 13–18.

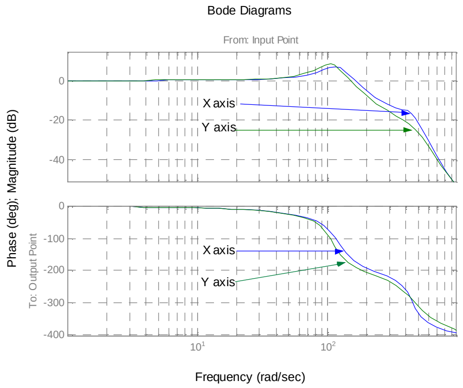

Fig. 12. Frequency response of the X and Y axis

♦ *Square 0.1 x 0.1 m (figures 13, 14):*

- For the spline function, the follow-up is good, except in coins where overshoots equal 5 mm. The path-time is 0.93 s.

- For the IGD jerk filter, the follow-up is very good with overshoots lower than 1 mm. The path-time is 30% longer with 1.2 s.

♦ *Square 0.01 x 0.01 m (figures 15, 16):*

- For the spline function, the follow-up is not good, especially in coins where overshoots are greater than 2 mm. The path-time is of 0.36 s.

- For the IGD jerk filter, the follow-up is better with overshoots lower than 0.2 mm but the path-time is longer 1.2 s.

♦ *Circle ∅=0.01 m (figures 17, 18):*

- For the spline function, the follow-up is very bad. The path-time is 0.45 s.

- For the IGD jerk filter, the follow-up is better with overshoots equal 0.2 mm but the path-time is longer 1.4 s.

The IGD jerk filter method, which takes into account a supplementary dynamic constraint, follows the path better than the spline function method in all examples, but does take longer. This little extra time, due to the extra constraint, ensures the better respect of the required path. With both methods, particularly the spline function, unfinished paths can be observed.

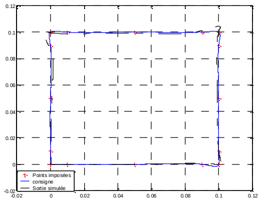

Fig. 13. Square 0.1m, Spline

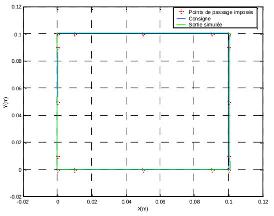

Fig. 14. Square 0.1m, DGIJ

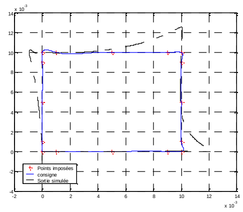

Fig. 15. Square 0.01m, Spline

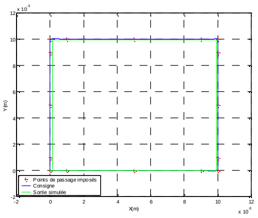

Fig. 16. Square 0.01m, DGIJ

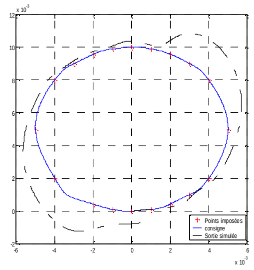

Fig. 17. Circle 0.01m, Spline

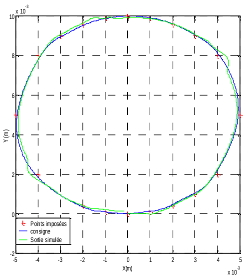

Fig. 18. Circle 0.01m, DGIJ

5. CONCLUSION

Motion control by an implicit generalized derivative jerk filter allows optimization (intermediate time optimization with static and dynamic constraints) in joint space.

This method uses the advantages of the Bang Bang law and the digital filter. It can be considered as an extension of the Bang Bang law, with the advantage of being simple: defined by only two synthesis parameters, and implemented in the form of a digital filter.

The IGD jerk filter method follows the path better than the spline function method but does take longer. This little extra time ensures the better respect of the required path. The method could be improved (precompensation of the integral gap between the reference input and the end actuator.

Experience shows that how many path points are chosen, and where they are placed, has a significant effect on the quality of the resulting path. This needs further investigation.

Also, the incomplete paths generated, even by the IGD jerk filter, could be compensated for perhaps by including an algorithm to modify the jerk input reference value.

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