Fractional Adaptive Control

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1. Introduction

- Fractional Calculus : A very old concept .... and a new research field
- These last years, many research works have been developped on the application of FC in various scientific fields ...
  - Mechanical systems:(Bagley:1991,1984,Makroglou:1994)
  - Electrical systems: (Le Méhauté:1983, Oldham:1983, Westerlund:1994)
  - Application in Biomedicine:(Ferdi:2003)
  - Heat transfer:(Loiseau:1998)
- Many authors have showed that fractional order derivative based models are often more adequate than the classical integer order models. (Caputo:1969, Nonnenmacher et Glöckle:1991, Friedrich:1991, Westerlund:1994)
- Fractional order derivatives and integrals give also a powerful tool for description of hareditary and mnemonic effects of various susbstances, and dynamical process modelization in fractal geometry (Mandelbrot:1982).

1. Introduction

- Fractional Order Control Systems

- Fractional order dynamical systems were studied only in a marginal way because of the lack of mathematical methods, both in control system theory and application.

- The idea of using factional order regulators for dynamical systems control was first proposed by Oustaloup, hwo developped the famous **CRONE "Commande Robuste d'Ordre Non Entier"** with examples and applications in various fields. He showed in particular the advantage of CRONE regulator when compared to classical PID.

- Podlubny (1999) has proposed later the $\mathbf{PI}^{\lambda}\mathbf{D}^{\mu}$ fractional order controller, using fractional order operators. He also showed that its performance are much better than those of integer order PID.

1. Introduction

- **Aim of this work :**
- Introduce fractional order operators in adaptive control algorithms of dynamical systems, in order to improve their performance
- Compare these fractional order control schemes behaviour with classical schemes one
- Study and proove stability of such control systems

2. About Fractional Calculus

Some Definitions :

- A single pole fractional order system

$$X(s) = \frac{k}{(1+\frac{s}{p})^{\beta}} \tag{1}$$

- A multiple pole fractional order system

$$X(s) = \frac{k}{\prod_{i=1}^{n}(1+\frac{s}{p_i})^{\beta_i}} \quad 0 \leq \beta_i \leq 1 \tag{2}$$

- A fractional order system of "‘second order"’

$$G(s) = \frac{1}{(\frac{s^2}{\omega_n^2} + 2\xi\frac{s}{\omega_n} + 1)^{\beta}} \tag{3}$$

2. About Fractional Calculus

Fractional order system performance

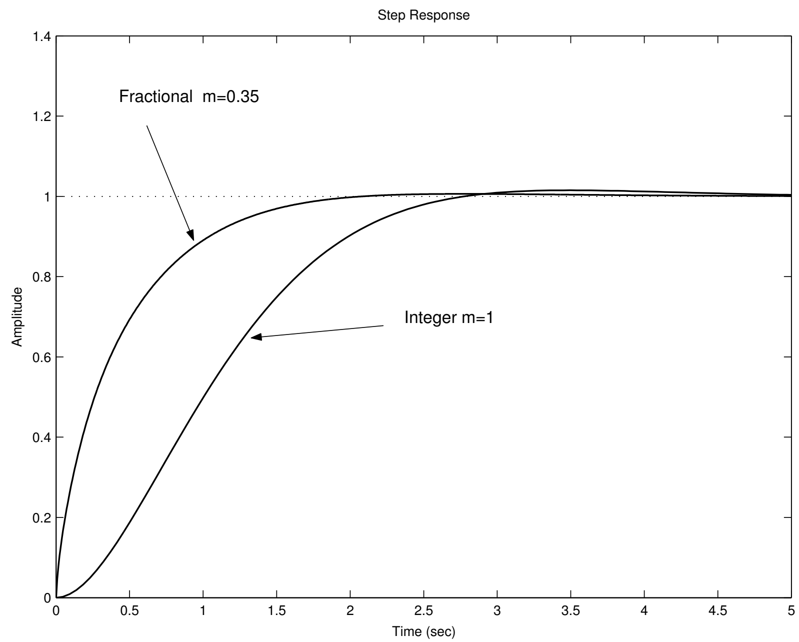

Figure 1: Comparative step responses of second order systems (Fractional/Integer)

2. About Fractional Calculus

- Fractional order Operators

- Riemann-Liouville Definition
- Integral:

$$ {}_{RL}I_{t_0}^{\lambda} f(t) \equiv \frac{1}{\Gamma(\lambda)} \int_{t_0}^{t} (t-\xi)^{\lambda-1} f(\xi) d(\xi) \tag{4} $$

- Derivative

$$ {}_{RL}D_{t_0}^{\mu} f(t) = \frac{1}{\Gamma(n-\mu)} \frac{d^n}{dt^n} \int_{t_0}^{t} (t-\tau)^{n-\mu-1} f(\tau) d\tau \tag{5} $$

where the integer n is such that $(n-1) < \mu < n$.

2. About Fractional Calculus

- Gründwald-Leitnikov Definition
- Derivative:

$$ {}_{GL}D^{\mu}f(t) = \frac{d^{\mu}}{dt^{\mu}}f(t) = \lim_{h\to 0} h^{-\mu}\sum_{j=0}^{k}(-1)^j \begin{pmatrix}\mu\\ j\end{pmatrix} f(kh-jh) \qquad (6) $$

- $\omega_j^{(\mu)} = \begin{pmatrix}\mu\\ j\end{pmatrix} = \frac{\Gamma(\mu+1)}{\Gamma(j+1)\Gamma(\mu-j+1)}$
- where $\omega_0^{(\mu)} = \begin{pmatrix}\mu\\ 0\end{pmatrix} = 1$, are the following binomial coefficients :

$$ (1-z)^{\mu} = \sum_{j=0}^{\infty}(-1)^j \begin{pmatrix}\mu\\ j\end{pmatrix} z^j = \sum_{j=0}^{\infty}\omega_j^{(\mu)} z^j \qquad (7) $$

2. About Fractional Calculus

- Charef's transfer approximation method: Singularity Fonction
- $$G(s) = \frac{1}{(1+\frac{s}{p_T})^{\beta}} \approx \frac{\prod_{i=0}^{N-1}\left(1+\frac{s}{z_i}\right)}{\prod_{i=0}^{N}\left(1+\frac{s}{p_i}\right)} \tag{8}$$
- where

$$p_i = (ab)^i p_0 \quad i = 1, 2, 3, ..., N \tag{9}$$

$$z_i = (ab)^i a p_0 \quad i = 2, 3, ..., N-1 \tag{10}$$

- and $p_0 = p_T 10^{\frac{\epsilon_p}{20\beta}}, \quad a = 10^{\frac{\epsilon_p}{10(1-\beta)}}, \quad b = 10^{\frac{\epsilon_p}{10\beta}}, \quad \beta = \frac{log(a)}{log(ab)}$

  ϵ_p tolerated error in dB

3. Fractional order MRAC

- Introduction

- One of the most popular adaptive control technics, developped by Whitaker et al. in 1958
- The desired performance are specified by a Reference Model
- An ordinary feedback + a supplementary feedback allowing regulator parameter ajustment based on the error between plant and model outputs
- Ajustment mechanism is based either on *the Gradient method* or *the stability theory*

3. Fractional order MRAC

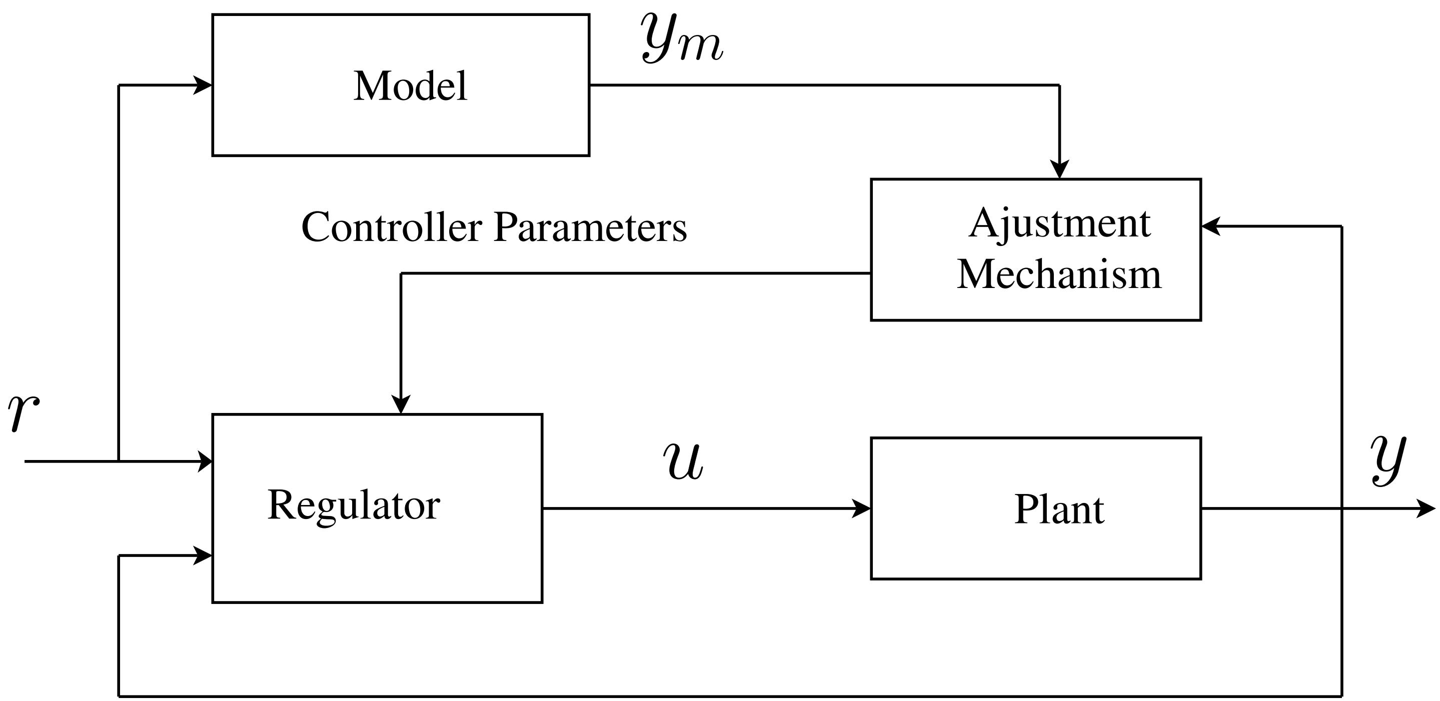

Figure 2: Model reference adaptive system (MRAS)

3. Fractional order MRAC

- M.I.T. rule
- $$J(\theta) = \frac{1}{2}e^2 \tag{11}$$
- To minimize J, the parameters are changed in direction of negative gradient of J,

$$\frac{d\theta}{dt} = -\gamma \frac{\delta J}{\delta \theta} = -\gamma e \frac{\delta e}{\delta \theta} \tag{12}$$

$$e(t) = \left(G_{BF}(p, \theta) - G_m(p)\right) r(t)$$

- $$\frac{d\theta}{dt} = \gamma \varphi e \tag{13}$$

3. Fractional order MRAC

- Regulator structure
- Plant model:

$$A\,y(t) = b_0 B\,u(t) \tag{14}$$

- Diophantine Equation:

$$Ru(t) = Tu_r(t) - Sy(t) \tag{15}$$

- Model error:

$$e = \frac{b_0}{A_o A_m}(Ru + Sy - Tu_r) \tag{16}$$

-

$$\theta^0 = (r_1 \dots r_k \; s_0 \dots s_l \; t_0 \dots t_m) \tag{17}$$

$$\varphi^T = \frac{b_0}{A_o A_m}\left(p^{k-1}u \dots u \; p^l y \dots y \; -p^m u_r \cdots - u_r\right) \tag{18}$$

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$$e = \varphi^T \theta^0 \tag{19}$$

3. Fractional order MRAC

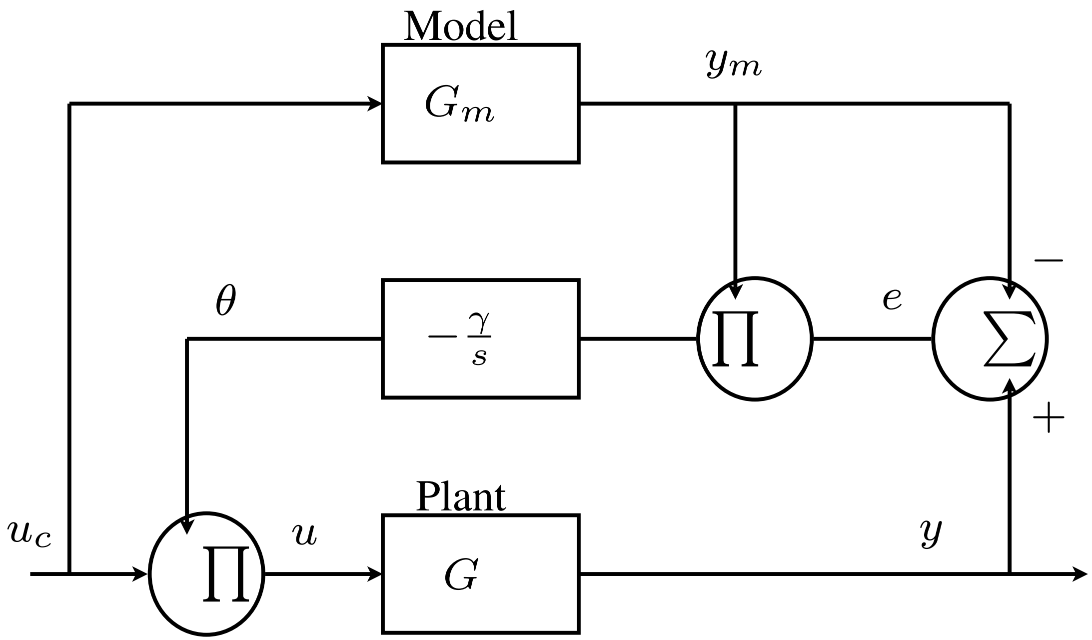

Figure 3: Classical Adaptation Algorithm.

3.1 Fractional Integration MRAC

- $$\theta = -\frac{\gamma}{s^\lambda} y_m (y - y_m) = -\frac{\gamma}{s^\lambda} y_m e$$

- $\Longrightarrow$
$$\frac{d^\lambda \theta}{dt^\lambda} = -\gamma y_m e \qquad (20)$$

- and,
$$\theta = -\gamma I^\lambda (y_m e) \qquad (21)$$

3.1 Fractional Integration MRAC

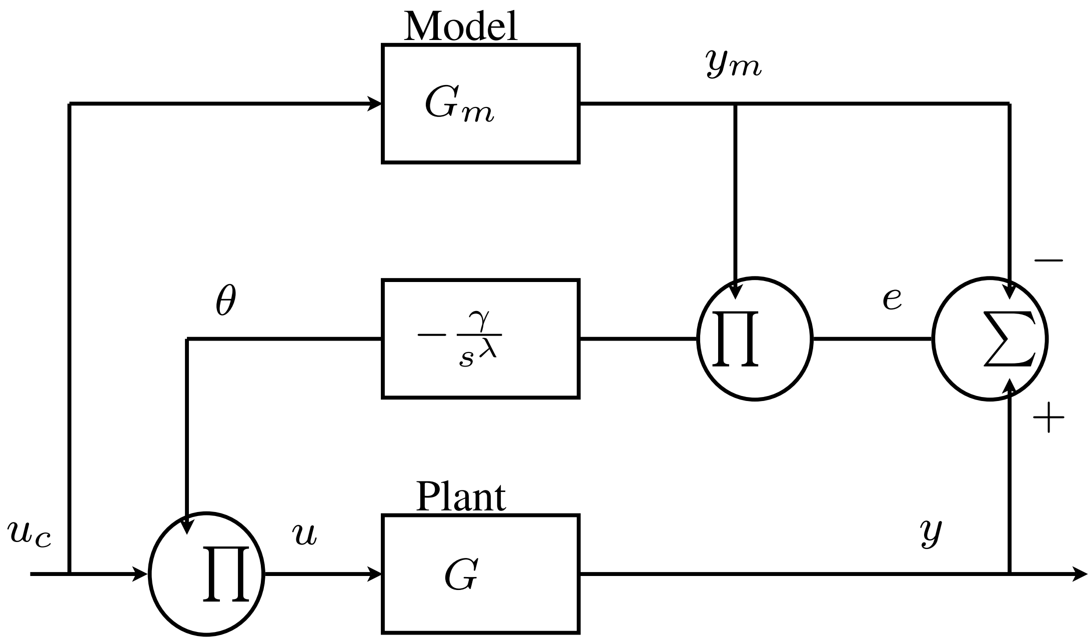

Figure 4: Fractional Integration Adaptive Algorithm.

3.1 Fractional Integration MRAC

- Simulation

- Plant model:

$$G(s) = \frac{81018}{s^2 + 260.7s + 2394} \quad (22)$$

- Reference Model

$$G_m(s) = \frac{100}{(s^2 + 9s + 100)^{0.4}} \quad (23)$$

3.1 Fractional Integration MRAC

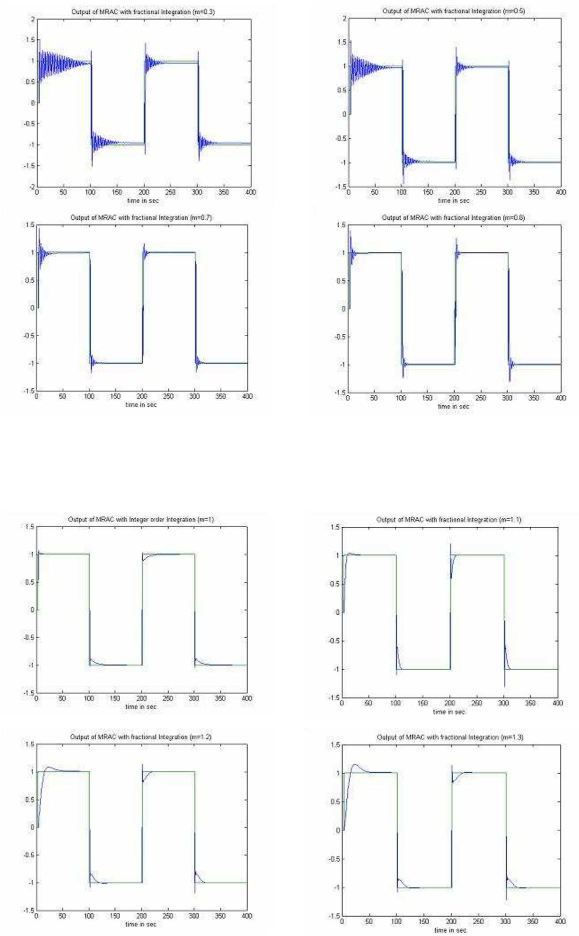

Figure 5: MRAC with FO Integration, order : $\lambda = 0.3 - 1.3$

3.1 Fractional Integration MRAC

- Remarks

- CL stability is achieved for all values of λ in the interval $]0, 2[$.
- The result obtained when $\lambda \approx 1.2$ is the best for output performance.
- For little values of integration order ($\lambda \leq 0.5$) the transitory phase presents oscillations.
- Vinagre et al. (2002) have noticed that the introduction of a fractional order integration in MRAC allows to enlarge the reference amplitude variation domain where the closed loop stability is maintained. Verified

3.2 Fractional Derivative MRAC

- The reference model G_m is a second order transfer function,
- We introduce a FO Derivative at the plant output, see Figure 6. We suppose that the relatif degree n of the siso plant is known,

  $G(s) = \frac{num(s)}{den(s)}$ with: $Deg(den(s)) - Deg(num(s)) = n$
- Then, when $|s|$ has a high value, we can write,

$$G_m(s) = \frac{y_m}{u_c} = \frac{1}{(\frac{s^2}{\omega_n^2} + 2\xi\frac{s}{\omega_n} + 1)^\beta} \approx_{|s|\to\infty} \frac{1}{s^{2\beta}}$$

- and also,

$$G(s) = \frac{y}{u} \approx_{|s|\to\infty} \frac{1}{s^n} \tag{24}$$

- In order compare $y_m(t)$ with $\frac{d^\alpha y(t)}{dt^\alpha}$ we must have,
  $\frac{s^\alpha}{s^n} = \frac{1}{s^{2\beta}}$ in other words:

$$\alpha = n - 2\beta \tag{25}$$

3.2 Fractional Derivative MRAC

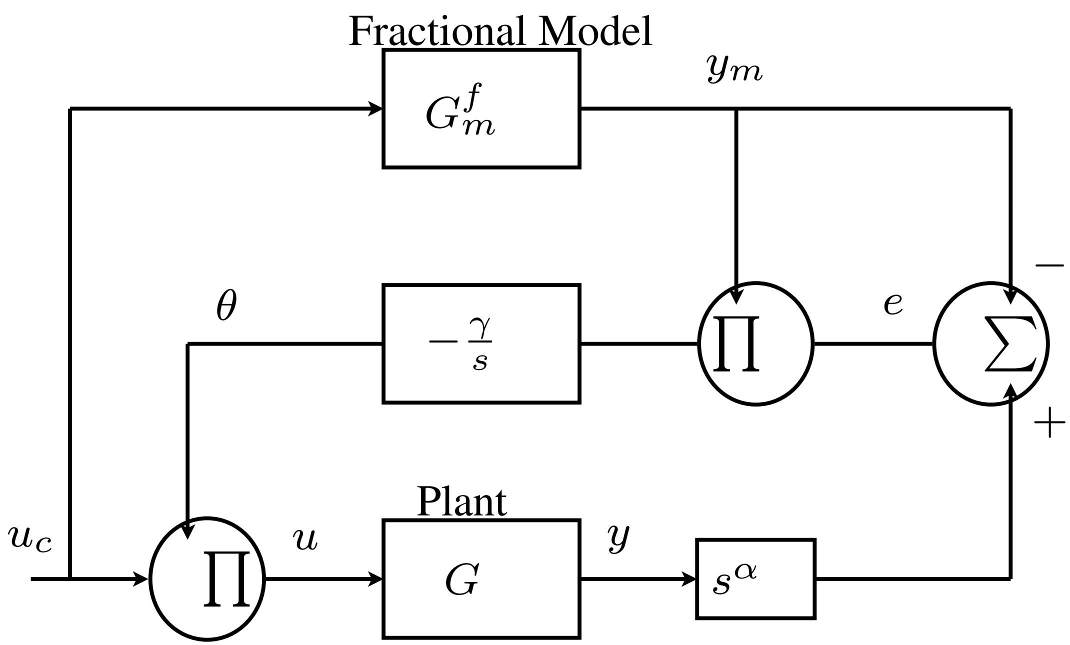

Figure 6: Adaptive algorithm with fractional order output derivative

3.2 Fractional Derivative MRAC

- Simulation
- Plant

$$G(s) = \frac{4 - 0.3\,s}{(s-0.2)(s+1)} \tag{26}$$

- Model of the integer order scheme

$$G_m^i(s) = \frac{2.25}{s^2 + 2.4s + 1} \tag{27}$$

- Model of the fractional order scheme

$$G_m^f(s) = \frac{2.25}{(s^2 + 2.4s + 1)^\beta} \tag{28}$$

- We obtain the results given in figures Figure 7 and Figure 8 for the case of integer order scheme and proposed fractional order scheme with $\beta = 0.35$ resp.,

3.2 Fractional Derivative MRAC

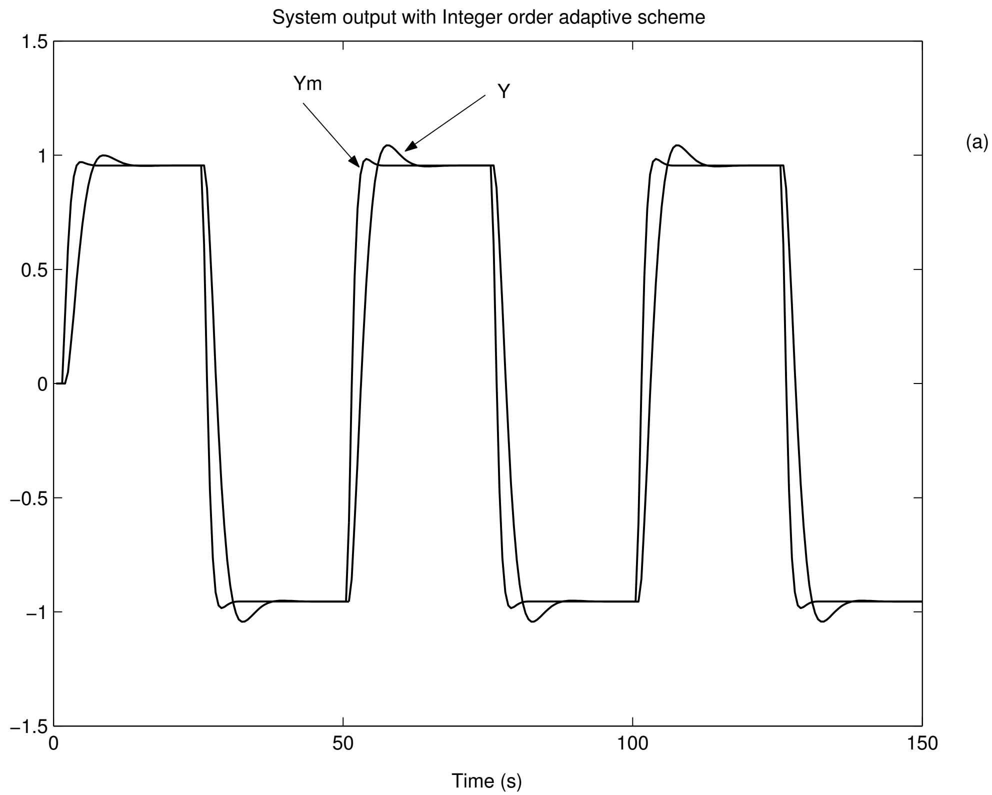

Figure 7: Plant output for integer order scheme

3.2 Fractional Derivative MRAC

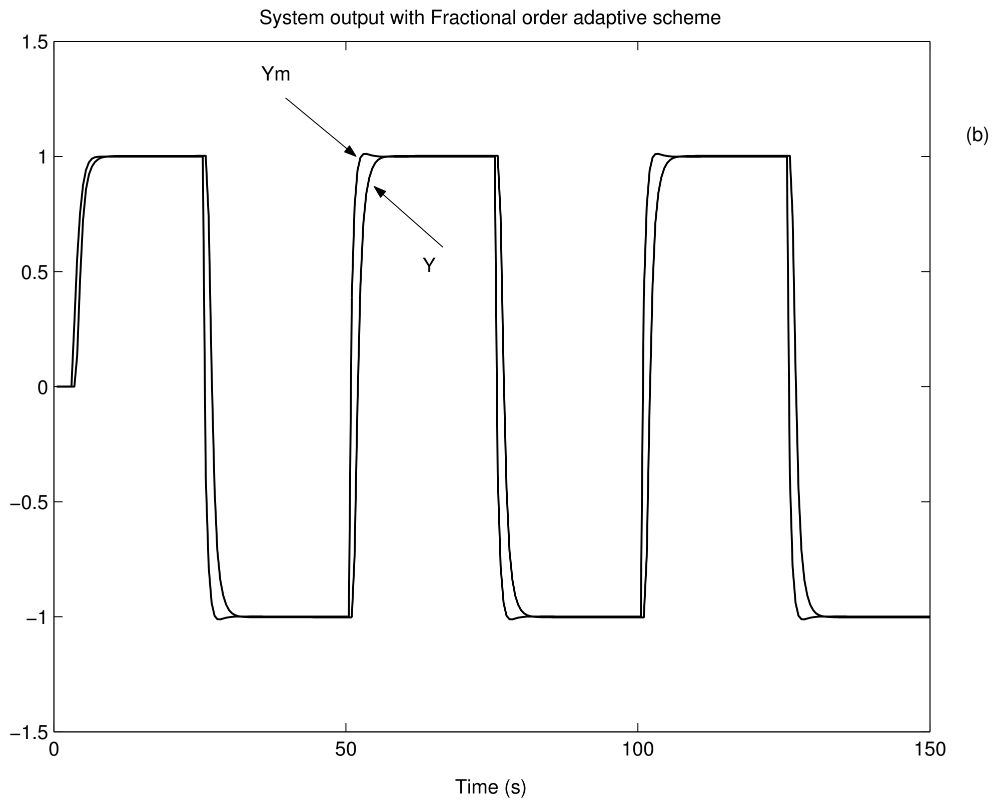

Figure 8: Plant output with fractional order scheme, $\beta = 0.35$

3.2 Fractional Derivative MRAC

- Remarks

- Stability is achieved for both cases, good level of performance.
- Robustness against perturbation is better in the fractional order scheme case.
- In fractional order scheme better responses are obtained for little values of adaptation gain γ (around 10^{-5})

3.2 Fractional Derivative MRAC

Remarks...

This is also confirmed with the quadratic error criterian J:

$$J = \sum_{k=0}^{N} (y(k\Delta) - u_c(k\Delta))^2 \tag{29}$$

Table 1: Quadratic error criterian

J	without perturbation	input perturbation	output perturbation
Integer	17.41	16.58	17.04
Fractional	8.97	9.08	9.11

3. Fractional order MRAC

- Conclusion

- A new fractional order model reference adaptive control algorithm for siso plant have been proposed
- CL stability is achieved with a good level of performance
- This appraoch is also interessant for fractional order systems control
- Analytical proof of stability is still an open problem

3. Fractional order MRAC

Publication

Journal :

1- S. Ladaci, A. Charef: "On fractional adaptive control", in Nonlinear Dynamics, Springer, Vol. 43, N°4. pp. 365-378. March 2006.

Conférences :

1- S. Ladaci, A. Charef : " MIT adaptive rule with fractional Integration " in Proceeding CESA'2003 IMACS Multiconference Computational Engineering in Systems Applications, Lille-France July, 9-11 2003.

2- S. Ladaci, A. Charef : " Model reference adaptive control with fractional derivative " in Proceeding CISTEMA'2003, 27-29 septembre 2003, Conference Internationale sur les systèmes de Télécommunication d'Electronique Médicale et d'Automatique, Université Aboubekr Blkaid, Tlemcen, Algérie.

3- S. Ladaci, J.J. Loiseau and A. Charef : "Using Fractional order Filter in Adaptive Control of Noisy plants". The 3rd Int Conf on Advances in Mechanical Engineering And Mecanics, ICAMEM 2006, December 17–19, Tunisia.

4. Adaptive PI$^\lambda$D$^\mu$ control

Introduction

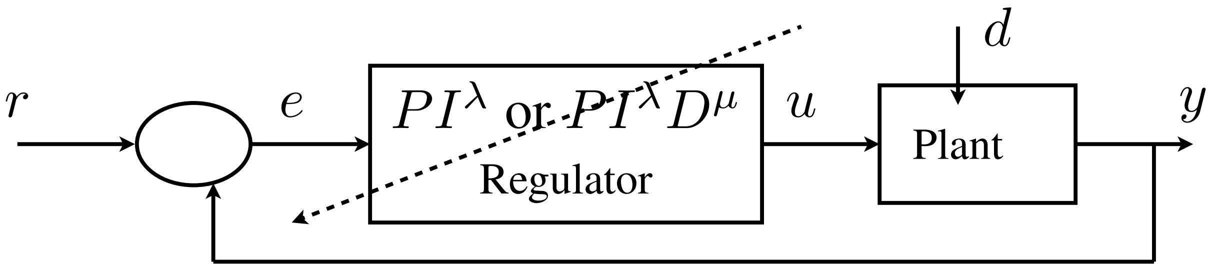

Figure 9: Adaptive PI^λ or $PI^\lambda D^\mu$ regulator of perturbed systems

4. Adaptive PI$^\lambda$D$^\mu$ control

- Introduction

- Principal contribution: introducing FO operators in a classical adaptive PI control scheme to obtain an adaptive PI$^\lambda$D$^\mu$ controller, with auto-tuning parameters, see Figure 9.
- This regulator is based on the high gain appraoch of Ilchmann et al 1993, and the simple adaptive PI controller of Fan J.C. et al 1998
- The interest in such regulator is justified by a best flexibility, because it adds new tuning parameters that are the fractional operators orders λ and μ

4. Adaptive $PI^{\lambda}D^{\mu}$ control

Control strategy

Original control scheme proposed by (Fan J.C., et al 1998) for minimum phase relatif degree one systems with constant perturbation :

$$u(t) = -k_c \left[k_1(t)e(t) + \int_0^t k_2(\tau)e(\tau)d\tau \right]$$

$$k_1(t) = k_p(t) + \alpha_1 k_i(t)$$

$$k_2(t) = \alpha_2 k_i(t)$$

$$k_p(t) = e^2(t) \tag{30}$$

$$k_i(t) = \int_0^t e^2(\tau)d\tau$$

$$e(t) = y(t) - r(t)$$

k_c, α_1 and α_2 are positive constants (tuning parameters).

4. Adaptive PI$^\lambda$D$^\mu$ control

...Proposed control scheme,

$$u(t) = -k_c \left[k_1(t)e(t) + I^\lambda(k_2(t)e(t)) + D^\mu(k_3(t)e(t))\right]$$

$$k_1(t) = k_p(t) + \alpha_1 k_i(t) + \alpha_3 k_d(t),$$

$$k_2(t) = \alpha_2 k_i(t)$$

$$k_3(t) = \alpha_4 k_d(t)$$

$$k_p(t) = e^2(t) \tag{31}$$

$$k_i(t) = I^\lambda(e^2(t))$$

$$k_d(t) = D^\mu(e^2(t))$$

$$e(t) = y(t) - r(t)$$

k_c,α_1, α_2, α_3 and α_4 are positive constants. The bloc diagram of the CL control system is shown in figure 10.

4. Adaptive PI$^\lambda$D$^\mu$ control

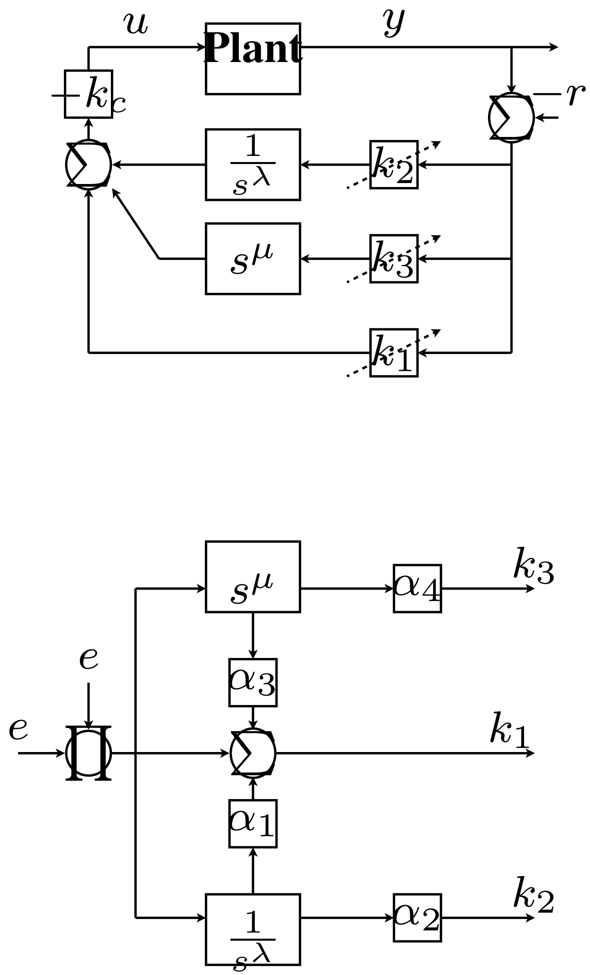

Figure 10: Fractional order adaptive $PI^\lambda D^\mu$ control system

4. Adaptive PI$^{\lambda}$D$^{\mu}$ control

Simulation

- We consider the example proposed by Fan J.C., et al 1998, to compare with the new regulation results:

$$\begin{aligned} \dot{x}(t) &= A\,x(t) + B\,u(t) + d \\ y(t) &= C\,x(t) \end{aligned} \tag{32}$$

- where d is a constant perturbation vector and:

$$A = \begin{pmatrix} -3 & 0 & 0 \\ 1 & 2 & -1.414 \\ 0 & 1.414 & 0 \end{pmatrix},\ B = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \tag{33}$$

$$C = \begin{pmatrix} 1 & 5 & 0 \end{pmatrix} \text{ and } d^T = \begin{pmatrix} 0.5 & 0.5 & .08 \end{pmatrix}$$

4. Adaptive PI$^{\lambda}$D$^{\mu}$ control

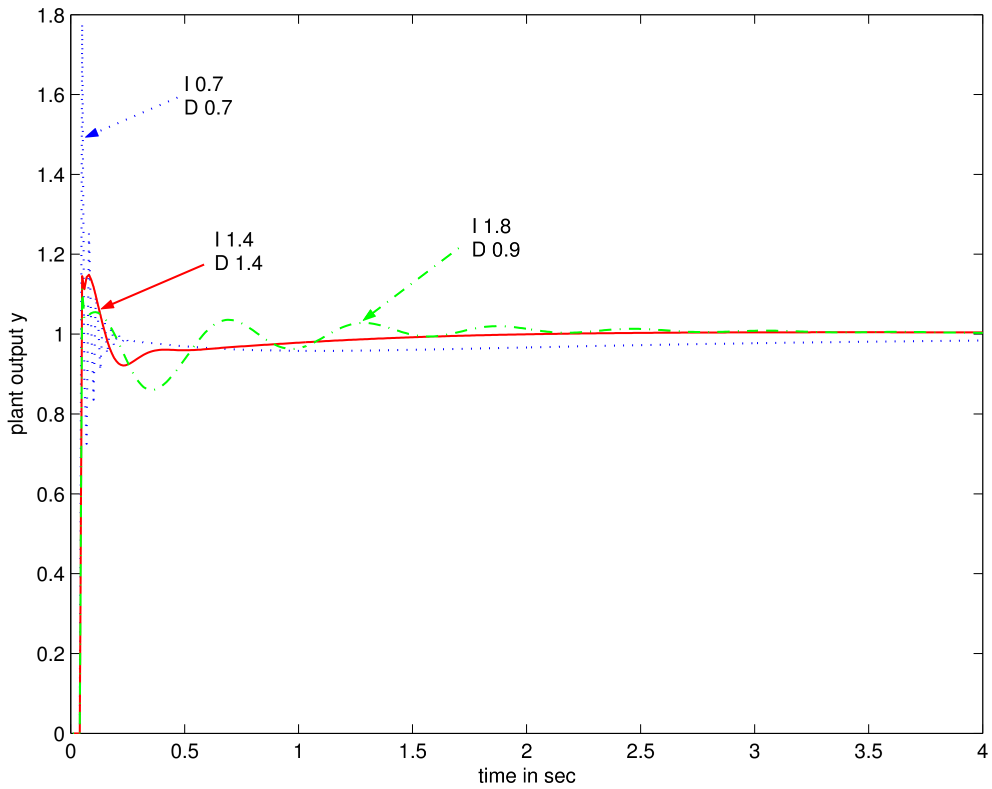

Figure 11: Output response for different values of the couple (λ, μ)

4. Adaptive PI$^\lambda$D$^\mu$ control

Remarks

- All initial values of the plant and regulator parameters are set to zero.
- Parameter tuning: First, we choose α_1, α_2 values (related to integration), and set α_3, α_4 to zero,

  Then, α_3, α_4 (related to derivative) are ajusted to improve CL performance
- Constant Perturbation rejection and reference following are well achieved for all λ and μ values,
- λ and μ have a great influence on the overall control system response time
- When λ is close to 2 the output response becomes oscillatory.

4. Adaptive PI$^\lambda$D$^\mu$ control

Remarks ...

To obtain the best fractional adaptive controller PI$^\lambda$D$^\mu$, we considered an objective function:

$$J = \int_0^T e^2(t)dt \tag{34}$$

Figure 12 shows the objective function J versus fractional integration order λ.
the least value of J is obtained for the couple $(\lambda, \mu) = (1.5, 1.2)$ with parameters $k_c = 17$,$\alpha_1 = 550$, $\alpha_2 = 35000$, $\alpha_3 = 10 - 4$, $\alpha_4 = 10 - 5$.
In this case the output response is given in Figure 13 where the overshoot is around 12%, stabilization time is less then $1s$.

These performance are much better than those obtained with Fan, J.C., et al, for the samr example.

4. Adaptive PI$^\lambda$D$^\mu$ control

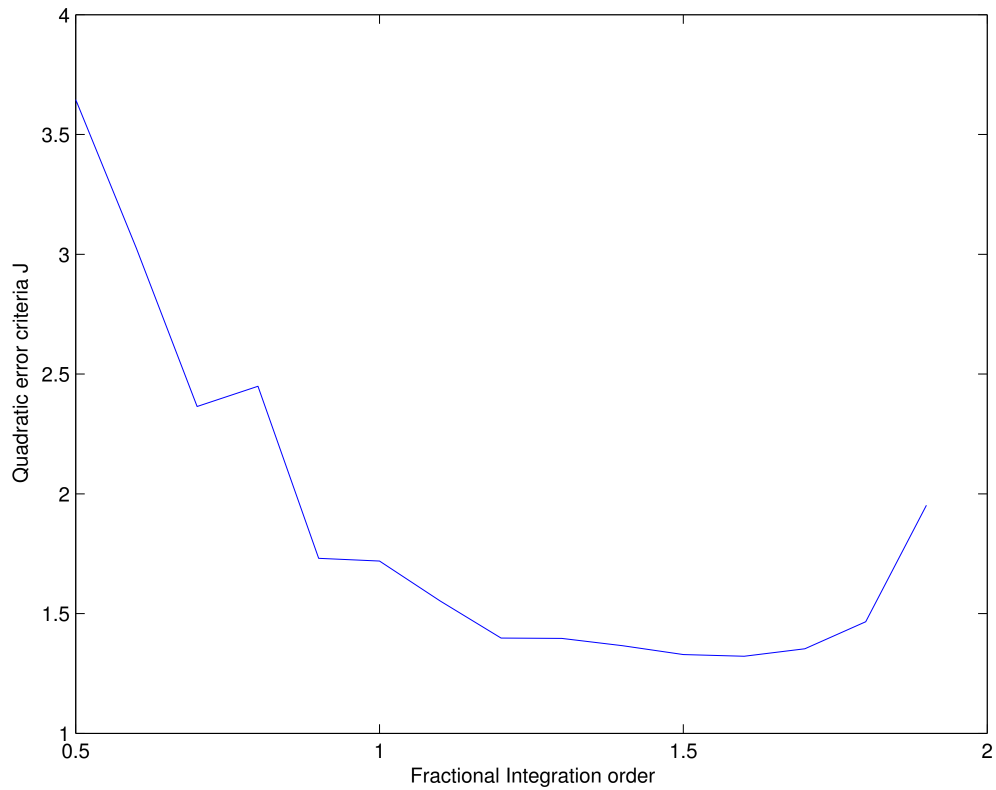

Figure 12: Quadratic error function versus intagration order λ

4. Adaptive PI$^{\lambda}$D$^{\mu}$ control

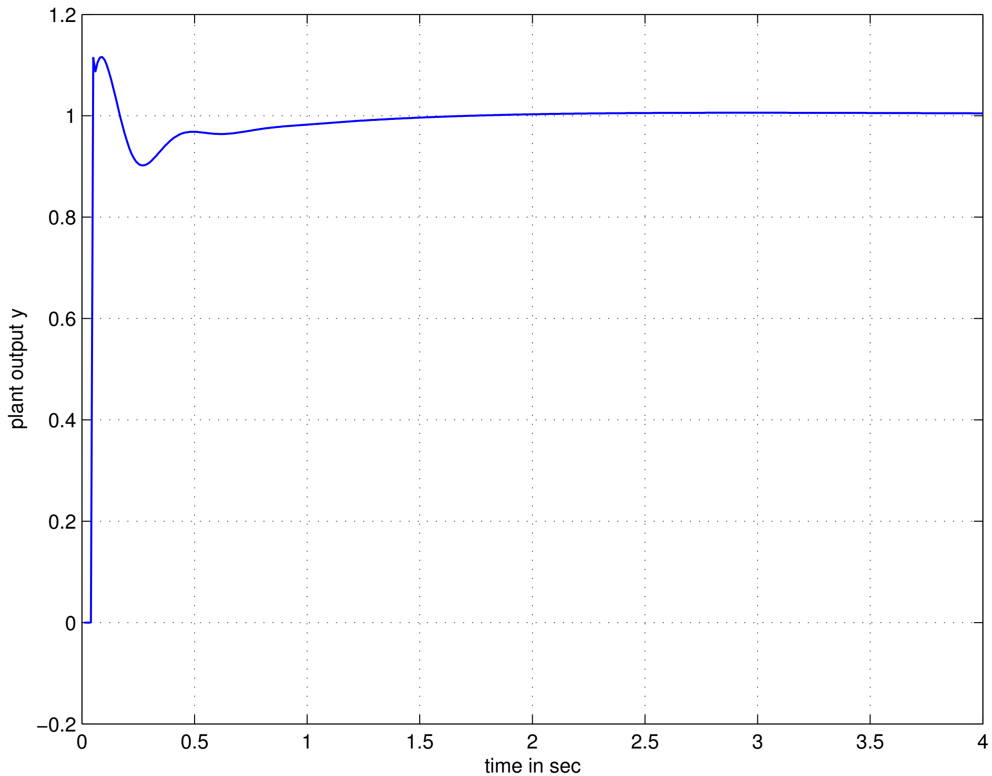

Figure 13: Output response for the couple $(\lambda, \mu) = (1.5, 1.2)$ $(k_c = 17, \alpha_1 = 550, \alpha_2 = 35000, \alpha_3 = 10^{-4}, \alpha_4 = 10^{-5})$

4. Adaptive PI$^{\lambda}$D$^{\mu}$ control

Conclusion

- The interest of this control scheme is to propose supplementary tuning parameters, that can improve the control system performance
- Perturbation rejection and reference following are well achieved, with better performance than those obtained with the intger order adaptive PI controller
- More research is to be done for applying such controller to a more general class of systems
- The simple construction of this regulator is a good argument when compared to classical invariant PI$^{\lambda}$D$^{\mu}$ controller (Podlubny)

4. Adaptive PI$^{\lambda}$D$^{\mu}$ control

Publication

Conference :

- S. Ladaci, A. Charef : " An Adaptive Fractional PIλDμ Controller " in Proceedings Sixth International Symposium on Tools and Methods of Competitive Engineering, TMCE 2006. ISBN: 961-6536-04-4. Avril 18-22, Ljubljana, Slovenia, 2006, Edited by I. Horvath and J. Duhovnik, pp. 1533-1540.

5. Conclusion & Outlines

- Interest of such adaptive control algorithms
- More research must be done towards analytical study of this controllers, especially for stability proof
- From this point of vue, the result obtaind on stability proof of fractional high gain controller is a first step in this way (to be presented in IECON'06)
- Developpe new fractional order appraoches, in many other control fields
- Applying these regulators for the control of real plants, for validation

Fractional Adaptive Control

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Thank you