UNIVERSITE PARIS XII - VAL DE MARNE
Laboratoire Images, Signaux & Systèmes Intelligents
E.A. 3956

presented by:

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IMAGE THRESHOLDING BASED ON FRACTIONAL DIFFERENTIATION

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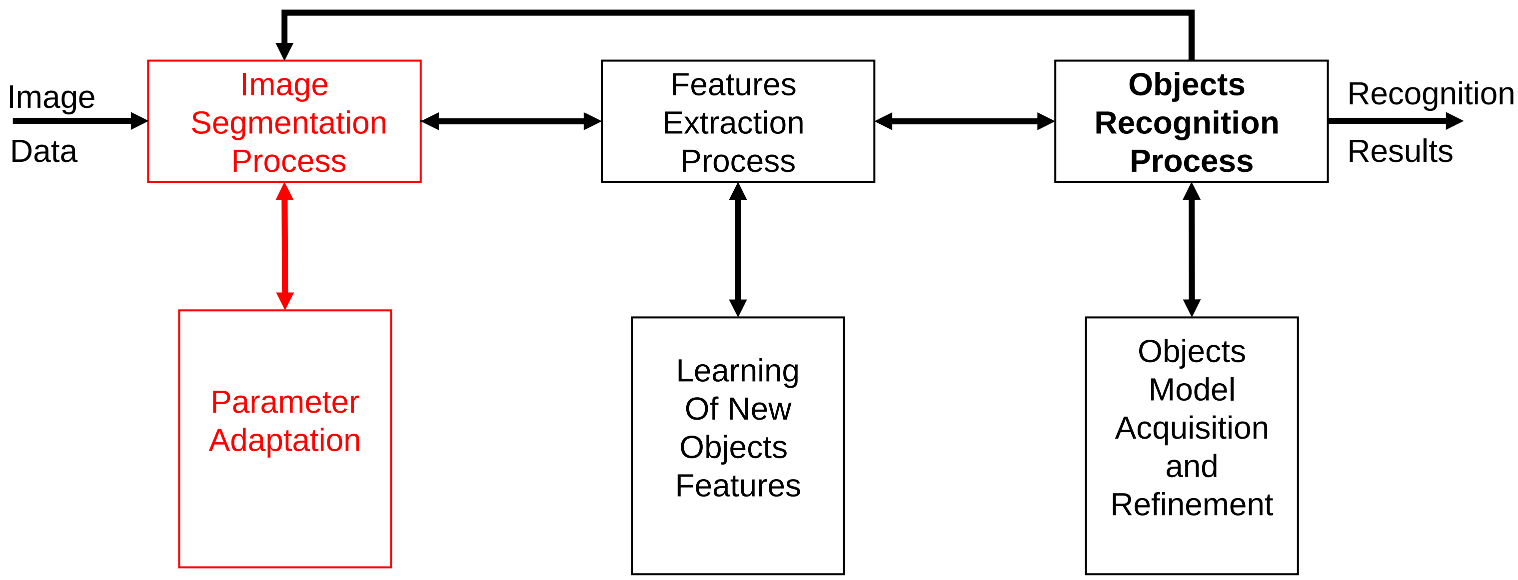
OUTLINE

1. Image segmentation problem
2. One-dimensional digital fractional differentiation (1D-DFD)
3. Two-dimensional digital fractional differentiation (2D-DFD)
4. Properties of fractional differentiated function
5. Proposed image thresholding methods based on 1D-DFD and 2D-DFD
6. Results and discussions
7. Conclusion

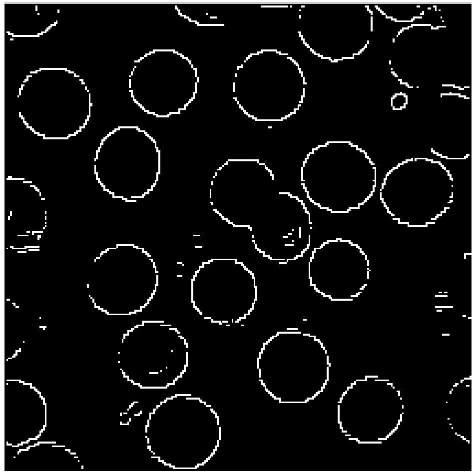


Conceptual design of multi-level computer vision system

- Segmentation goal : identify the regions of image objects and extract the objects from the background.
- None segmentation technique is able to produce consistently good results on a wide range of existing images, ….
- There is no universally accepted measure of segmentation performance available to uniquely define the quality of the segmented image. However, several measures are used that are sensitive to different image characteristics and features.

SEGMENTATION METHODS

Edge detection



Region detection

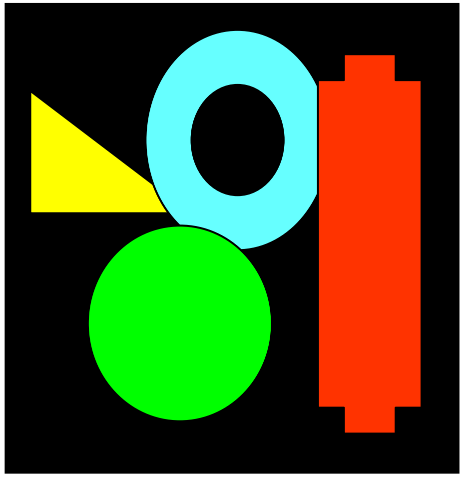


Image thresholding

Thresholding is a pixel classification technique based on the value of a single feature, usually pixel gray-level:

$$R(x,y) = k \quad \text{if} \quad T_{k-1} \le f(x,y) < T_k \quad \text{for} \quad k = 0,1,\ldots,m$$

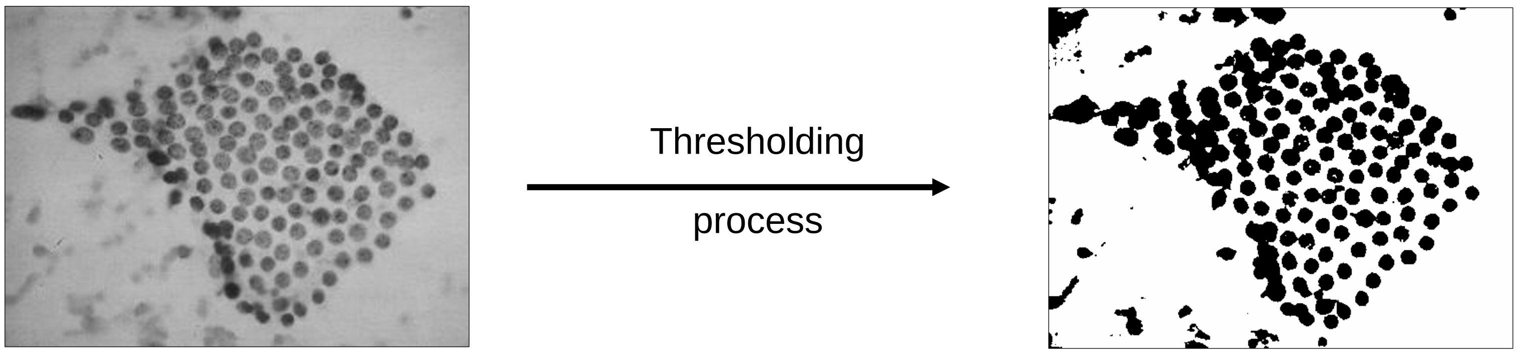
where

$f(x, y)$ and $R(x, y)$ are the gray-level and label of the pixel (x, y), respectively.

$T_0 < \ldots < T_m$ are thresholds.

m : the number of distinct labels assigned to segmented image.

Example: The obtained black regions correspond to cells. An example of image analysis, is cell counting.



Process of the image thresholding

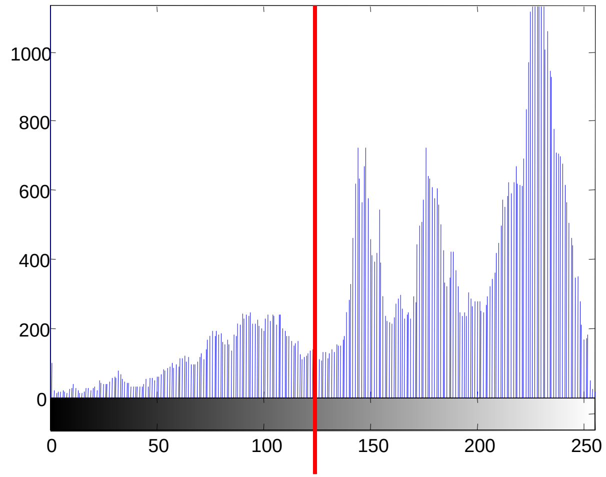
Manual thresholding:

- Image histogram
- ***Take thresholds in valleys***
- Labeling the different classes
- Pixel classification

How can we choose thresholds automatically?

EXAMPLE

global thresholding of an infrared image.



T=125

The Riemann-Liouville operator D^{α}, for fractional differentiation is defined by the formula:

$$D^{-\alpha} f(x) = \frac{1}{\Gamma(\alpha)} \int_{c}^{x} (x-\xi)^{\alpha-1} f(\xi)\, d\xi$$

where:

$f(x)$ a real causal function, $x > 0, \quad \mathrm{Re}(\alpha) > 0$, *c* the integral reference and Γ gamma function.

The fractional integration J^{α} can be computed by:

$$J^{\alpha} D^{\alpha} f = f, \quad m-1 \le \alpha \le m, \quad m \in \mathrm{N}$$

The discrete form of fractional differentiation (DFD) of order α presented by Grünwald is given by :

$$g(x) = D^{\alpha} f(x) = \frac{1}{h^{\alpha}} \sum_{k=0}^{M} \omega_k(\alpha) f(x-kh)$$

where *h* is sampling step, *M* is the number of samples and $\omega_k(\alpha)$ are the binomial coefficients:

$$\omega_0(\alpha) = 1 \qquad \omega_{k+1}(\alpha) = \frac{(k+1)-\alpha-1}{(k+1)} \omega_k(\alpha), \quad k = 0, 1, 2, \ldots, M-1$$

We consider the image as a 2D real bounded function f(x, y), the 2D-DFD is then given by the formula:

$$D^{\alpha} f(x,y) = \frac{1}{h^{2\alpha}} \sum_{k=0}^{M} \sum_{l=0}^{M} p(k,l).f(x-k, y-l)$$

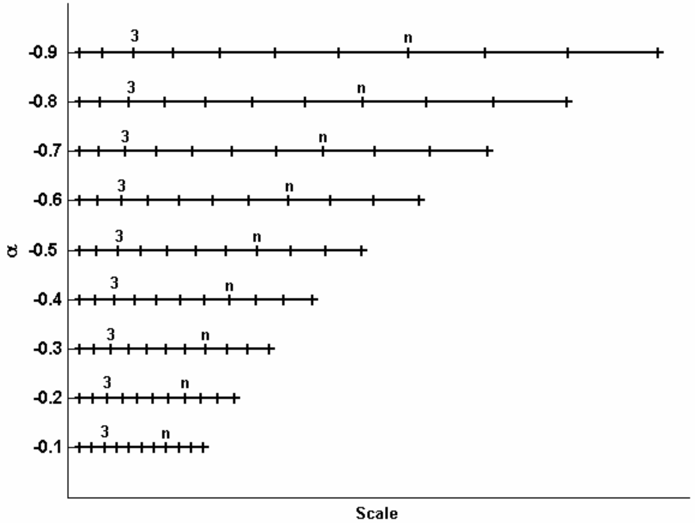
where: M+1 represents the number of past elements of f considered to calculate the fractional differentiated image, it also represents the size of the « mask ».

$p(k,l) = \omega_k(\alpha) \times \omega_l(\alpha)$ are elements of the matrix $P\big(p(k,l)\big)_{\substack{0 \le k \le M \\ 0 \le l \le M}}$

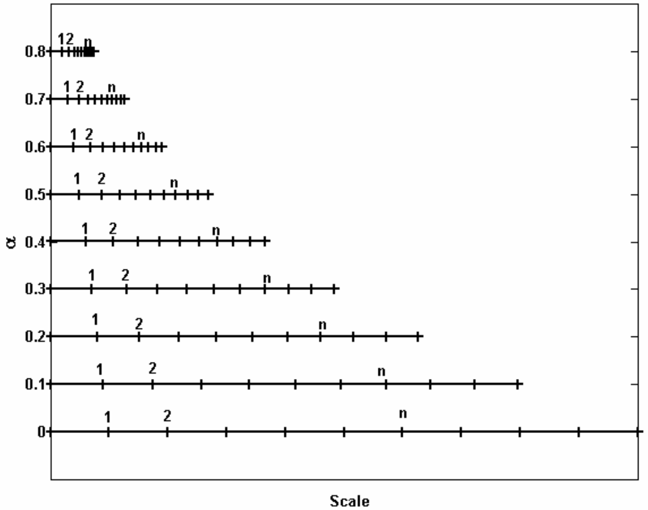
The previous equation can be seen as a convolution of the image f with the filter $P_M^{(\alpha)}(x,y)$

$$g(x,y) = D^{\alpha} f(x,y) = f(x,y) \otimes P_M^{(\alpha)}(x,y)$$

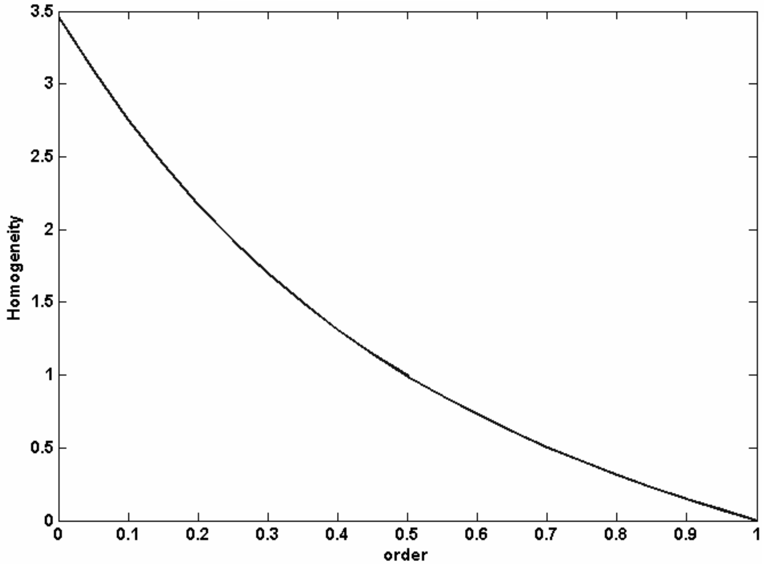
$\otimes$ 2D convolution operator

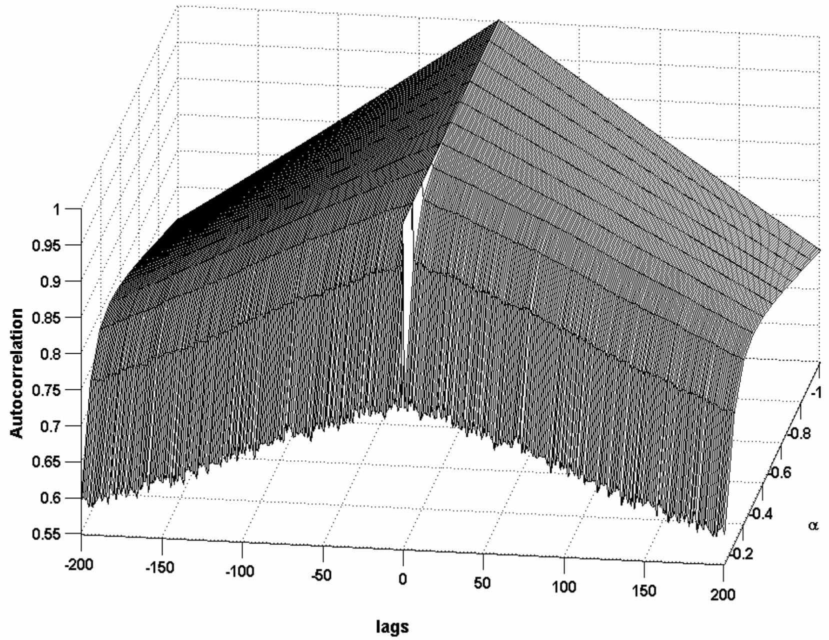


Negative order

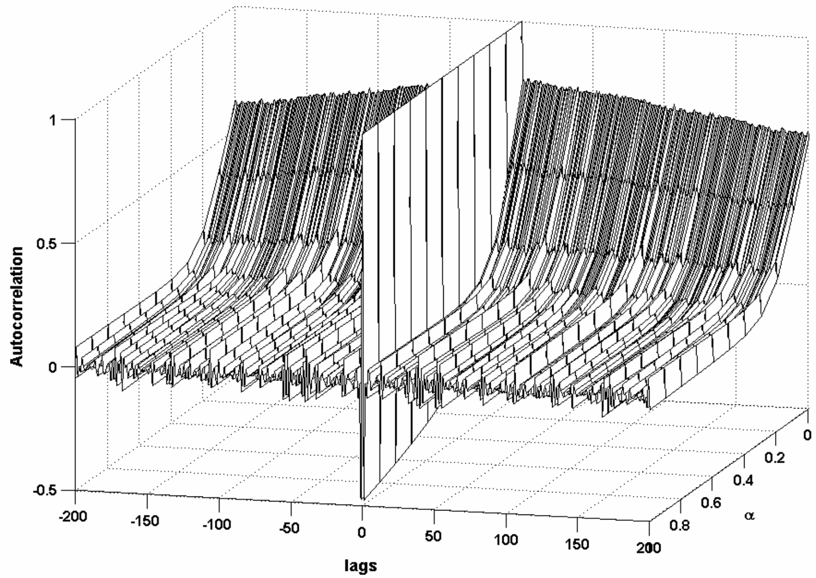


Positive order





Autocorrelation increasing with negative DFD order. Example of application of a DFD to a zero mean white noise signal.



Autocorrelation decreasing with positive DFD order. Example of application of a DFD to a zero mean white noise signal.

The 1D case

The Z transfer function of the discrete fractional differentiation filter can be written as: $H(z) = \frac{G(z)}{F(z)} = \left(1 - z^{-1}\right)^{\alpha}$

The average value of the output signal i.e. the differentiated function is given by: $\mu_g = H(0)\mu_f$

where μ_f is the average value of the function *f*. The value of μ_g is infinite when the DFD order is negative.

The 2D case

the function *g*(*x,y*) can be interpreted as the output function of a discrete filter, *f*(*x,y*) is the input function (image). The transfer function of this filter can be written as:

$$H(u,v) = \frac{G(u,v)}{F(u,v)} = \frac{1}{h^{2\alpha}} \sum_{k=0}^{M} \sum_{l=0}^{M} p(k,l).\exp^{-j2\pi(uk+ul)}$$

we can easily show that the average value of the output signal, or the differentiated function, is given by:

$$\mu_g = \mu_f.E\{H(u,v)\} = \mu_f.\frac{1}{h^{2\alpha}} \sum_{k=0}^{M} \sum_{l=0}^{M} p(k,l)$$

HYPOTHESIS

The basic idea of this technique of image segmentation was inspired from the assumption that there is a dependency between the different image pixels, and the value of each one is related to its neighbours.

To exploit the properties of fractional differentiation, seen above, and to apply them to image segmentation.

- *The histogram's autocorrelation is decreased in the case of non-noised images (consequently the pixels weakly correlated are eliminated) and the two classes strongly correlated are separated.*

- *In the case of noised images, we increase the histogram's autocorrelation (thus eliminating the noise) in order to separate the foreground from the background.*

- *We propose to decrease this correlation such as only pixels strongly correlated will appear in the 2D-DFD image.*

Algorithm 1 – Image thresholding algorithm based on 1D-DFD

1. Calculate the image histogram $h(x)$ to be segmented.

2. Calculate the DFD of $h(x)$ with a given order α, denoted by $D^{\alpha} h$

3. Define the threshold as follows:

Let be a set:

$$\Omega_{\alpha} = \{ i : i \in [p_1 \quad p_2] \text{and } D^{\alpha} h(i) > 0 \}$$

where p_1 and p_2 are the arguments of the two most significant peaks in the differentiated histogram, then the threshold is given by:

$$th_{\alpha} = 1\mathrm{I}_{\{\alpha \geq 0\}} \times Sup\ \Omega_{\alpha} + 1\mathrm{I}_{\{\alpha < 0\}} \times Min\ \Omega_{\alpha}$$

where $1\mathrm{I}_{\{\alpha \geq 0\}}$ is Heaviside function: equal to 1 if the condition $\alpha \geq 0$ is satisfied

4. Print results: order (α) , threshold (th) and thresholded image.

Algorithm 2 – Image thresholding algorithm based on 2D-DFD

1. Calculate the 2D-DFD of the image to be segmented with the given α, denoted by $D^{\alpha}I$.
2. Calculate $h^{(\alpha)}(x)$ the histogram of $D^{\alpha}I$.
3. Define the threshold as follows:

   Let be a set: $\Omega_{\alpha} = \{i : h^{(\alpha)}(i) > 0\}$, then the threshold is given by:

$$th_{\alpha} = 1\mathrm{I}_{\{\alpha \geq 0\}} \times Sup\,\Omega_{\alpha}$$

where $1\mathrm{I}_{\{\alpha \geq 0\}}$ is Heaviside function: equal to 1 if the condition $\alpha \geq 0$ is satisfied.

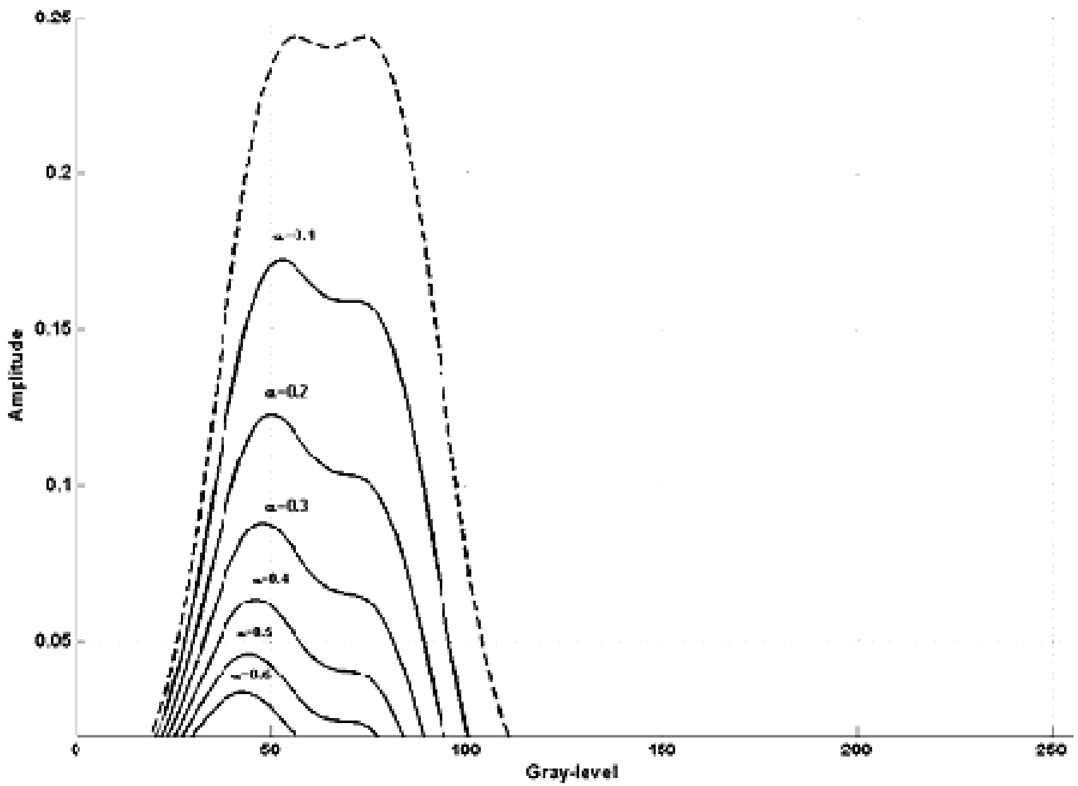
As Ω_{α} is a finite set, we can write:

$$th_{\alpha} = 1\mathrm{I}_{\{\alpha \geq 0\}} \times \max \Omega_{\alpha}\,.$$

4. Print results: 2D-DFD order (α), threshold (th_{α}) and segmented image.

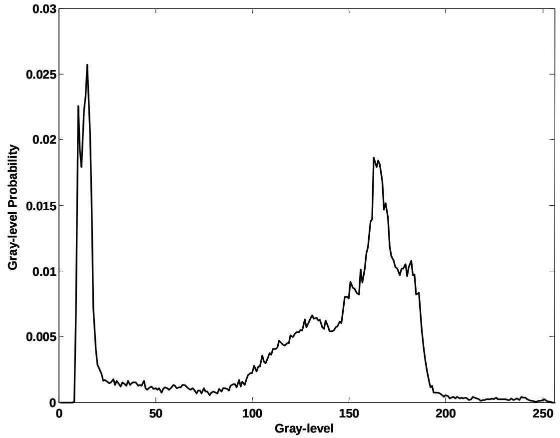
Example of image histogram

Decreasing of amplitude range with $\alpha>0$. Original causal and non negative function (dashed line) and fractional differentiated function (continuous line) with α varying from 0.1 to 0.6 (step=0.1).



ALGORITHM1- 1D-DFD	ALGORITHM2- 2D-DFD

(a)



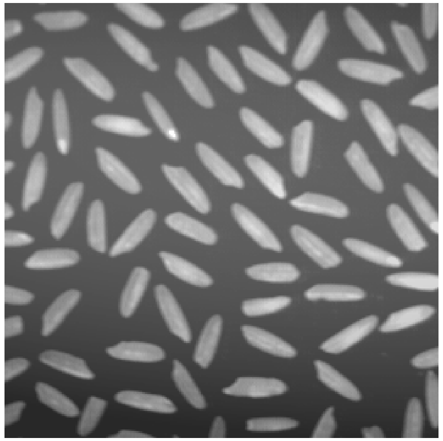
(b)

(c)

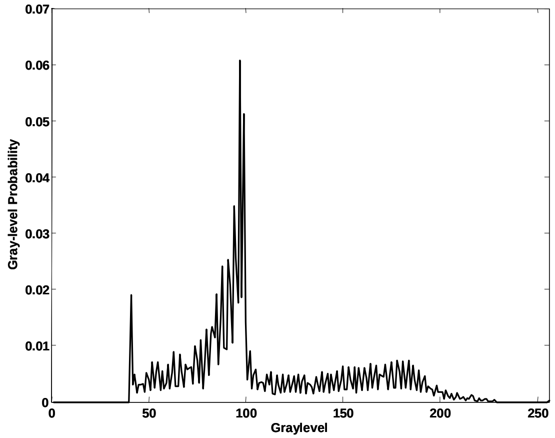
(d)

Cameraman image, its gray-level histogram and the thesholded images. (a) Original cameraman image, (b) image histogram, (c) segmented image (*th*=26, *α*=-0.3), (d) segmented image (*th*=79, α=0.3).

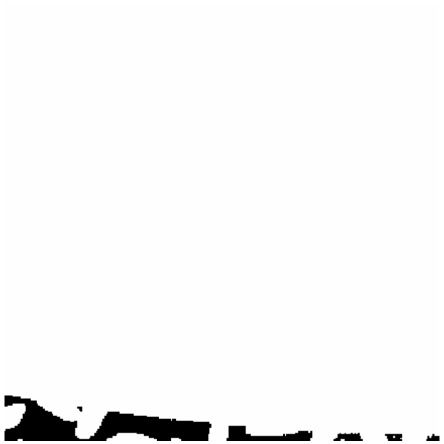
ALGORITHM1- 1D-DFD | **ALGORITHM2- 2D-DFD**



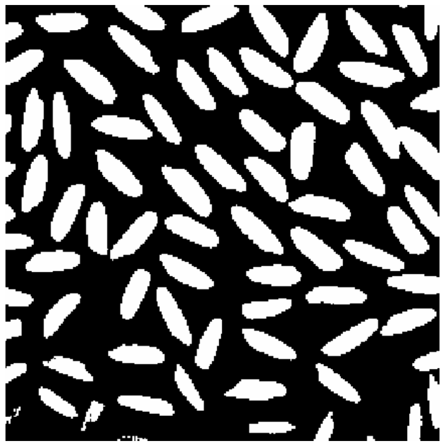
(a)



(b)



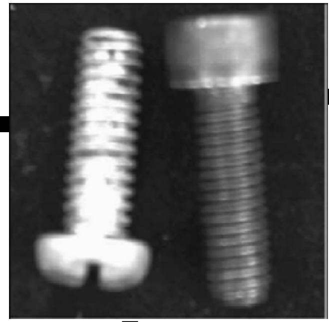
(c)



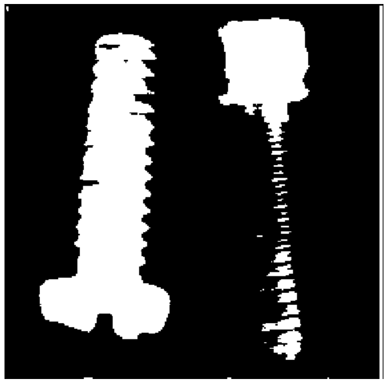
(d)

Rice image, its gray-level histogram and the thesholded images. (a) Original rice image, (b) image histogram, (c) segmented image (*th*=43, *α*=-0.3), (d) segmented image (*th*=113, α=0.3).

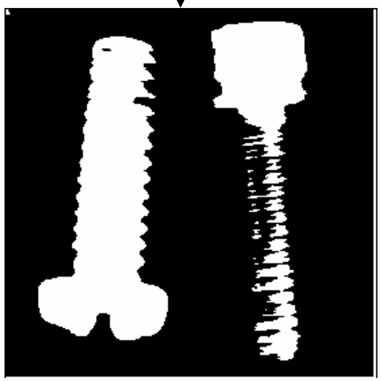
Original image



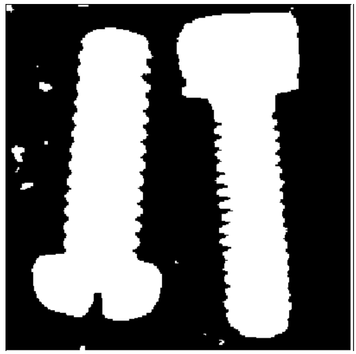
segmentation result through OTSU method



segmentation result through Kapur method

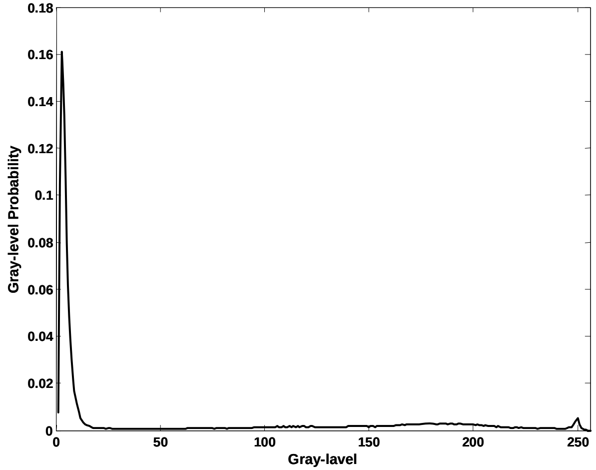


segmentation result through 1D-DFD method

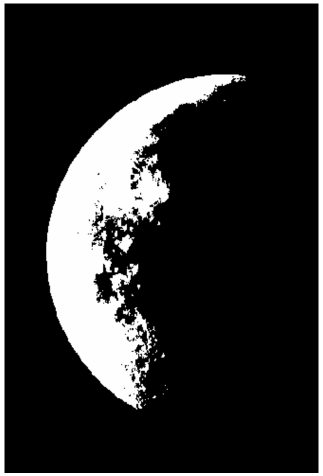


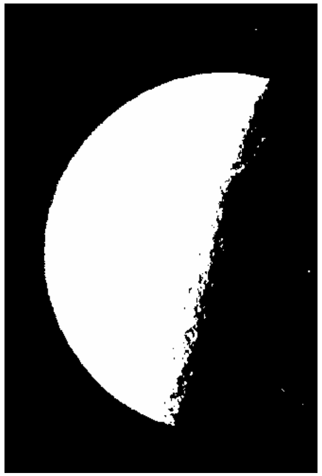
ALGORITHM1- 1D-DFD	ALGORITHM2- 2D-DFD


(a)



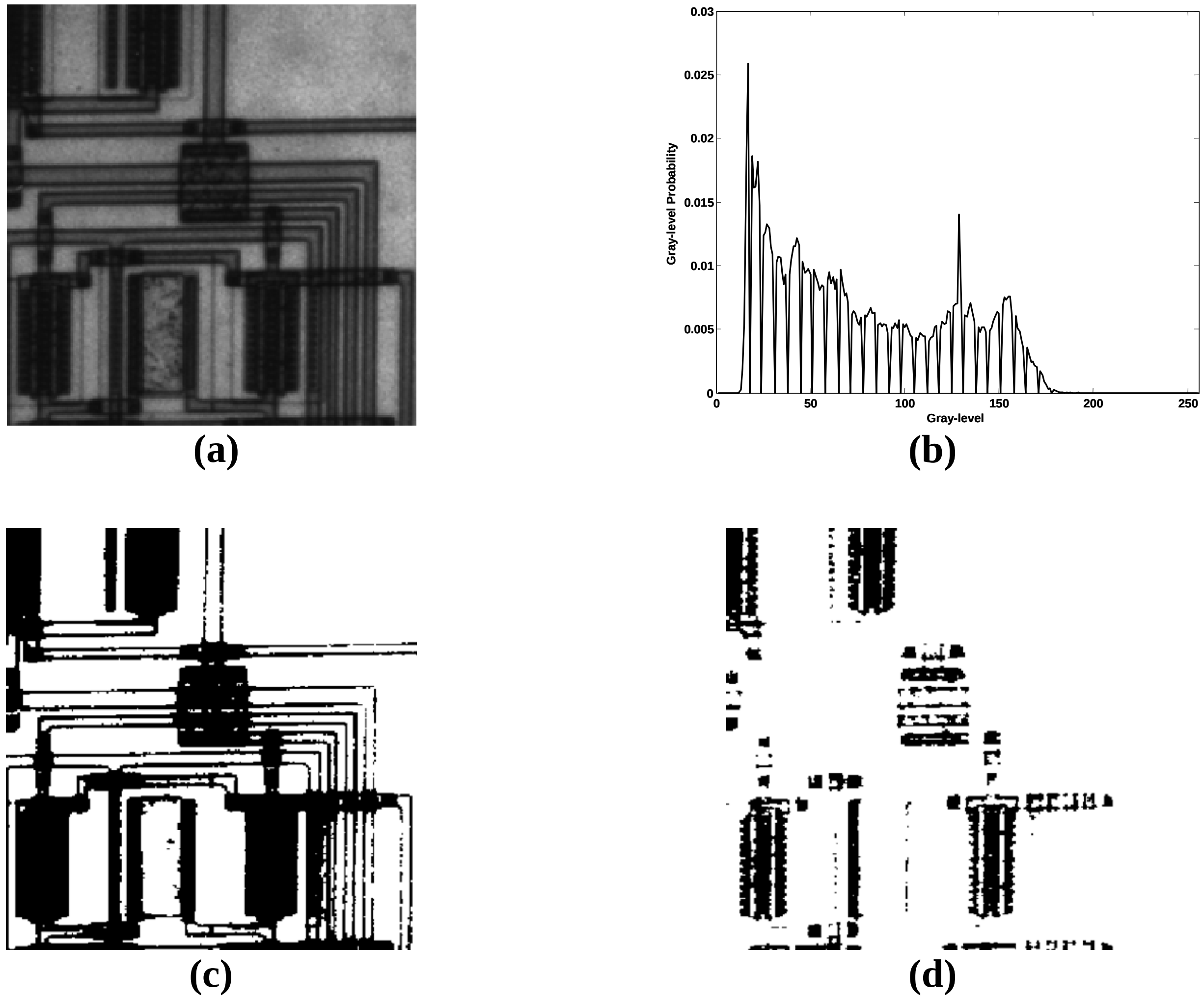
(b)


(c)


(d)

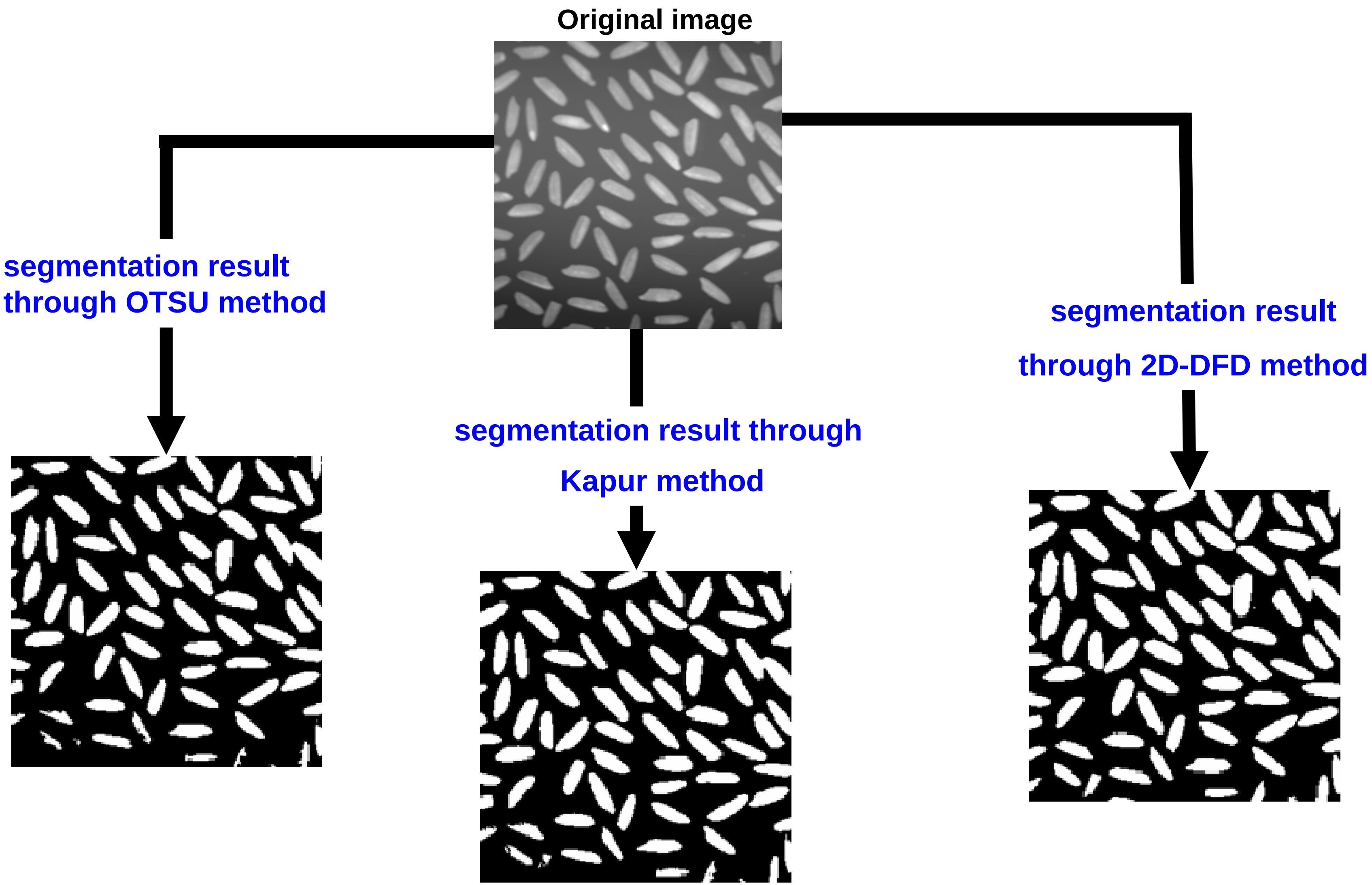
Moon image, its gray-level histogram and the thesholded images. (a) Original moon image, (b) image histogram, (c) segmented image (*th*=109, α=0.1), (d) segmented image (*th*=28, α=0.5).

ALGORITHM1- 1D-DFD	ALGORITHM2- 2D-DFD



Circuit image, its gray-level histogram and the thesholded images. (a) Original circuit image, (b) image histogram, (c) segmented image (*th*=50, *α*=0.3), (d) segmented image (*th*=23, α=0.5).

ALGORITHM1- 1D-DFD	ALGORITHM2- 2D-DFD



- We developed two methods for image segmentation that employ one-dimensional or two-dimensional digital fractional differentiation.

- The proposed method uses fractional differentiation with a positive order to decrease the correlation. Consequently, only the gray-levels strongly correlated will not be eliminated but will be separated. At the end, the gray-levels belonging to a same object will be detected.

- These new bilevel thresholding methods can be extended to multilevel thresholding by means of modifications of which study is under progress.

References

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THANK YOU FOR YOUR ATTENTION

Questions