Robust Adaptive Control Using A Fractional Feedforward Based on SPR Condition

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1. Introduction

- Adaptive control algorithms show their limits in noisy or disturbed environment, becoming inefficient or uncompetitive.
- The use of simple parallel feedforward in the adaptation loop appeared as a robust solution since the 80's. It can guarantee robust stability of the nonlinear adaptive controller. (Bar Kana)
- Main contribution of this work: A fractional robust adaptive control solution for disturbed applications, based on stabilizability property of the plant and simple parallel feedforward in order to satisfy the desired "almost positive realness" condition

2. Fractional order systems

A SISO fractional order system can be represented by the following transfer function,

$$X(s) = \frac{b_m s^{\beta_m} + b_{m-1} s^{\beta_{m-1}} + \ldots + b_0 s^{\beta_0}}{a_n s^{\alpha_n} + a_{n-1} s^{\alpha_{n-1}} + \ldots + a_0 s^{\alpha_0}} \qquad (1)$$

Where,

- α_i, β_j: real numbers such that,

$$\begin{cases} 0 \leq \alpha_0 < \alpha_1 < \ldots < \alpha_n \\ 0 \leq \beta_0 < \beta_1 < \ldots < \beta_m \end{cases}$$

Definition 1 *The fractional order transfer function Matrix $M_X(s)$ whose elements are of the form (1) is proper (strictly proper) if and only if all elements of $M_X(s)$ are bounded at ∞ (tend to zero at ∞, resp.).*

Fractional systems present best qualities, in response time and in transition dynamic stability, and robustness in presence of uncertainties and perturbations.

3. Concept of Positive Realness Condition

Definition 2 *The $m \times m$ transfer function matrix $G_s(s)$ is called strictly positive real (**SPR**) if:*

1. *All elements of $G_s(s)$ are analytic in $\Re(s) \geq 0$.*
2. *$G_s(s)$ is real for real s.*
3. *$G_s(s) + G_s^{T*}(s) > 0$ for $\Re(s) \geq 0$ and finite s.*

We can show that for a fractional order transfer function matrix $G_s(s)$,

$$G_s(s) \text{ is } \mathbf{SPR} \Leftrightarrow G_s^{-1}(s) \text{ is } \mathbf{SPR} \qquad (2)$$

Definition 3 *Let $G_a(s)$ be a $m \times m$ transfer matrix. Let us assume that there exists a positive definite constant gain matrix, $\tilde{K}_e$ such that the closed-loop transfer function*

$$G_c(s) = \left[I + G_a(s)\tilde{K}_e\right]^{-1} G_a(s) \qquad (3)$$

*is **SPR**. $G_a(s)$ is called "almost strictly positive real (**ASPR**)".*

3. Concept of Positive Realness Condition

Now if we consider a fractional order proper or strictly proper **ASPR** transfer matrix $G_s(s)$. Then the following statements are equivalent,

$$G_s(s) = [I + G_a(s)K_e]^{-1}\, G_a(s) \text{ is SPR} \tag{4}$$

$$G_s(s) = [I + G_a(s)K_e]^{-1} \text{ is SPR} \tag{5}$$

$$G_s^{-1}(s) = G_a^{-1}(s) + K_e \text{ is SPR} \tag{6}$$

$$\Re\left[G_a^{-1}(s) + K_e\right]_{\Re(s)\geq 0} > 0 \tag{7}$$

$$\begin{array}{ll} G_s^{-1}(s) & \text{is asymptotically stable and} \\ K_e & \text{is sufficiently large} \end{array} \tag{8}$$

3. Concept of Positive Realness Condition

Because $\exists M$ such that $\Re\left[G_a^{-1}(s)\right]_{\Re(s)\geq 0} > M > -\infty$, and then any $K_e > -M$ will do.

$$\begin{array}{rl} G_a(s) & \text{is strictly minimum phase and} \\ K_e & \text{is sufficiently large} \end{array} \qquad (9)$$

Here we can generalize as fellows the result of (Bar-Kana 1989) to the fractional order case.

Lemma 1 *Let a fractional order transfer function matrix $G_a(s)$ be* **ASPR** *and let $\tilde{K}_e$ be any gain that satisfies (3). Then $G_a(s)$ is* **SPR** *for any gain K_e that satisfies $K_e > \tilde{K}_e$.*

It is obvious that **ASPR** fractional order systems, which are minimum phase proper systems maintain stability with high gains.

3. Concept of Positive Realness Condition

Remarks

1. The ASPR plant must also be proper.
2. The open loop is not necessarily stable (the plant will actually be stabilized by the fictitious gain K_e), however all the zeros must be placed in the left half plane. The plant must be minimum phase to obtain positivity.
3. We can easily show that if a system is ASPR, then it can be stabilized by any constant or time variable output gain K_e, if it is large enough, i.e. $K_e > \tilde{K}_e$.

But in this method, instead of using high gain regulation we will use a simple parallel feedforward configuration which can by a similar way satisfy the positive realness conditions.
The idea of using feedforward in parallel with the controlled plant is based on the following Lemma,

Lemma 2 *(Bar-Kana 1989) Let the plant be described by the $m \times m$ transfer function $G_p(s)$ of order n. Let $C(s)$ be any dynamic stability output feedback controller. Then*

$$G_a(s) = G_p(s) + C^{-1}(s) \tag{10}$$

is **ASPR** *if $C^{-1}(s)$ is proper or strictly proper.*

4. Main Result

We propose a fractional order feedforward configuration of the form:

$$F(s) = \frac{F_p}{\left(1 + \frac{s}{s_0}\right)^{\alpha}} \tag{11}$$

with a real fractional power $0 < \alpha < 1$, to improve the robustness of the adaptive algorithm, in presence of perturbations, as such systems do not amplify much these random signals.

Theorem 1 *Let $G(s)$ be any $m \times m$ strictly proper transfer matrix of arbitrary MacMillan degree. $G(s)$ is not necessarily stable or minimum phase. Let*

$$H_f(s) = K(1 + qs^{\alpha}) \tag{12}$$

be some stabilizing controller for $G(s)$. Then the augmented controlled plant

$$G_a^f(s) = G(s) + H_f^{-1}(s) = G(s) + \frac{K^{-1}}{1 + qs^{\alpha}} \tag{13}$$

is **ASPR**.

4. Main Result

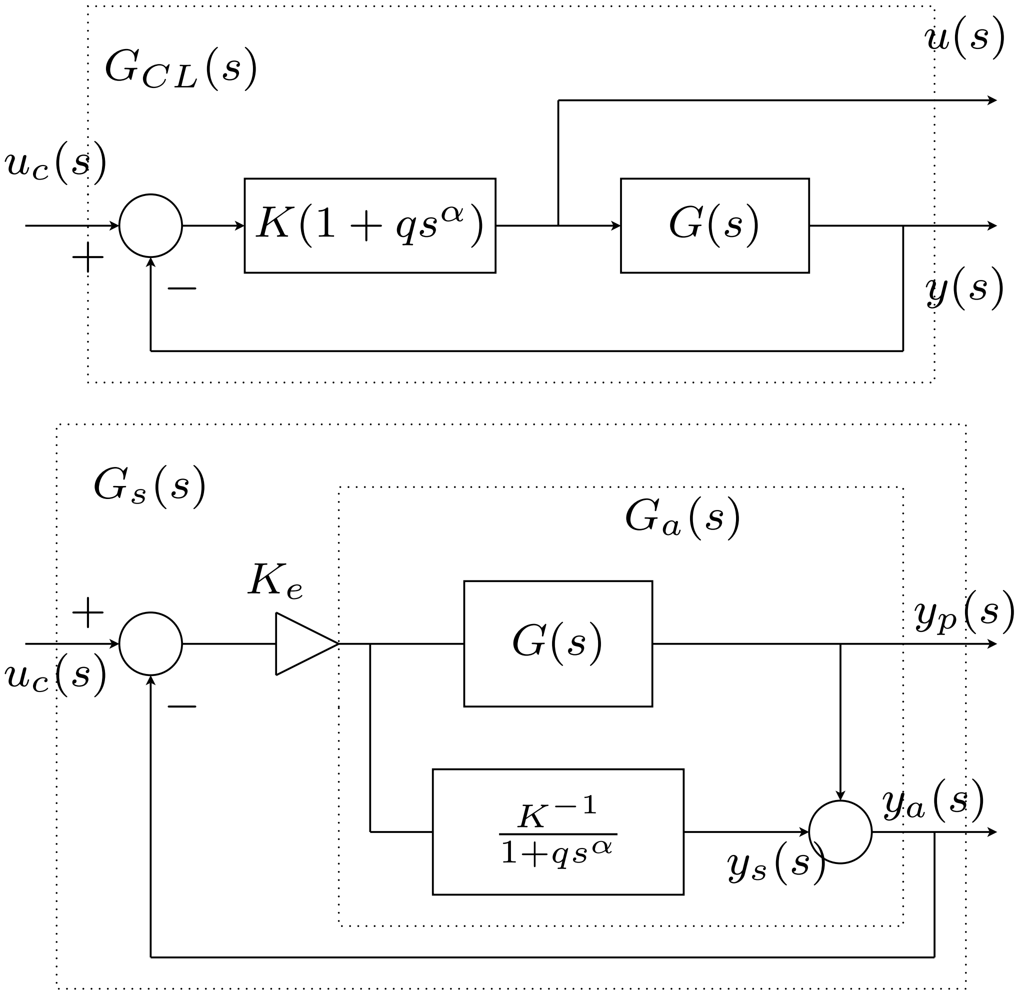


Figure 1: The fictitious SPR configuration.

4. Main Result

The stabilizing controller $H_f(s)$ can also be modelized as follows,

$$H_f(s) = K(1 + qs)^{\alpha} \tag{14}$$

From Definition 3 and the fact that the transfer function $G_a^f(s)$ is ASPR, we know that it can be stabilized by a gain $\tilde{K}_e$. Figure 1 illustrates the feedforward configuration. In addition, the stabilization is robust, it holds for any gain $K_e > \tilde{K}_e$.

Many previous works (Hotzel 1997, Podlubny 1999) have proposed PD$^{\mu}$ improper controllers of the form (12):

$$C(s) = K_p + K_i s^{\alpha} \tag{15}$$

which can stabilize many realistic plants for sufficient high values of K.

4. Main Result

A feedforward of equivalent effect is chosen as follows:

$$F(s) = C^{-1}(s) = \frac{F_p}{\left(1 + \frac{s}{s_0}\right)^{\alpha}} \tag{16}$$

Where $F_p = K^{-1}$, such that the augmented plant becomes,

$$G_a(s) = G_p(s) + F(s) \tag{17}$$

As K should be very large, so F_p are small coefficients, guaranteeing that $G_a(s)$ be **ASPR**. And during the control design we can take $G_a(s) \approx G_p(s)$ as a practical approximation.

5. Implementation in MRAC scheme

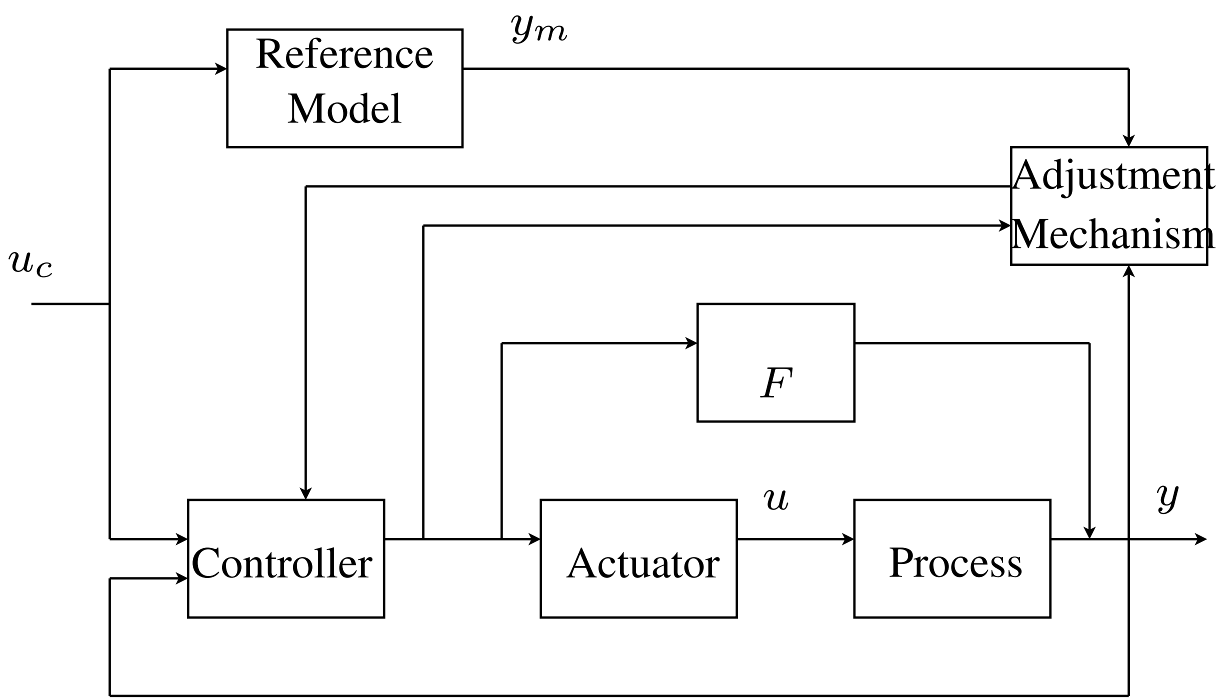


Figure 2: Simple feedforward in MRAC scheme.

5. Implementation in MRAC scheme

Model Reference Adaptive Control (MRAC) is one of the more used approaches of adaptive control, in which the desired performance is specified by the choice of a reference model. Adjustment of parameters is achieved by means of the error between the output of the plant and the model reference output.

We consider a closed loop system where the controller has an adjustable parameter vector θ. A model which output is y_m specifies the desired closed loop response. Let e be the error between the closed loop system output y and the model one y_m, one possibility is to adjust the parameters such that the cost function:

$$J(\theta) = \frac{1}{2}e^2 \tag{18}$$

be minimised.

$$\frac{d\theta}{dt} = -\gamma\frac{\delta J}{\delta\theta} = -\gamma e\frac{\delta e}{\delta\theta} \tag{19}$$

or

$$\frac{d\theta}{dt} = \gamma\varphi e \tag{20}$$

This aproach is called *M.I.T. rule*.

6. Simulation Example

Without any loss of generality we will apply this robust adaptive control method, both in the case of integer and fractional order feedforward, to a SISO model of a DC motor controlled in respect of velocity, given by:

$$G_p(z) = \frac{0.8513z + 5.099\ 10^{-6}}{z^2 + 2.442\ 10^{-7}z + 1.37\ 10^{-11}} \tag{21}$$

with a sampling period $T_s = 0.3sec$, and an actuator model of the form:

$$A(z) = \frac{0.007667z + 0.007049}{z^2 - 1.763z + 0.7772} \tag{22}$$

The plant is subject to random input and output perturbations of amplitudes 2 and 0.05 respectively.

The reference model G_m is given by:

$$G_m(z) = \frac{0.9411z + 0.1208}{z^2 + 0.05679z + 0.005092} \tag{23}$$

6. Simulation Example

1. Integer order feedforward case The feedforward trunsfer fuction F is given by:

$$F(z) = \frac{3.2394\ 10^{-7}}{z - 0.9997} \qquad (24)$$

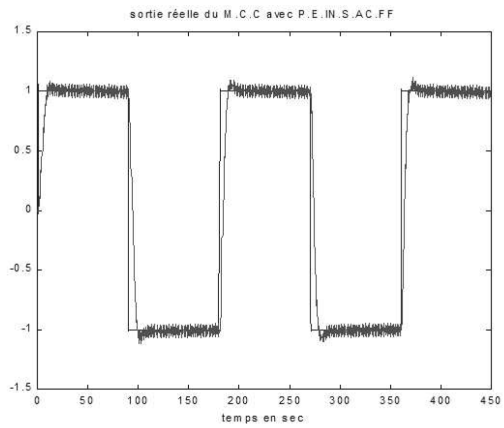
with a regulation parameter $\gamma = 0.001$ we obtain the results of Figure 3.
2. Fractional order feedforward case The fractional order feedforward trunsfer function F is given in Laplace domain by:

$$F(s) = \frac{0.001}{(s + 500)^{0.6}} \qquad (25)$$

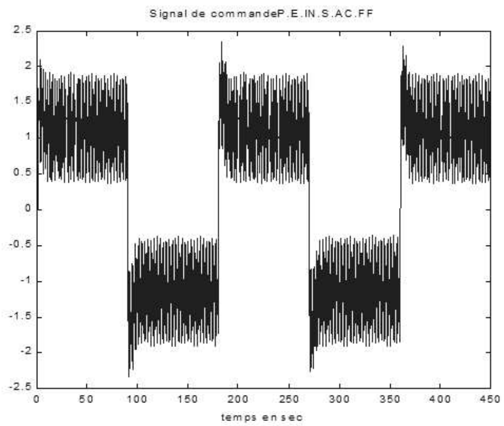
For the purpose of our approach we need to use an integer order model approximation of the fractional order feedforward model in order to implement the adaptation algorithm, for this aim we have used the so-called singularity function method.

with a regulation parameter $\gamma = 0.005$, we obtain the results of Figure 4.

6. Simulation Example



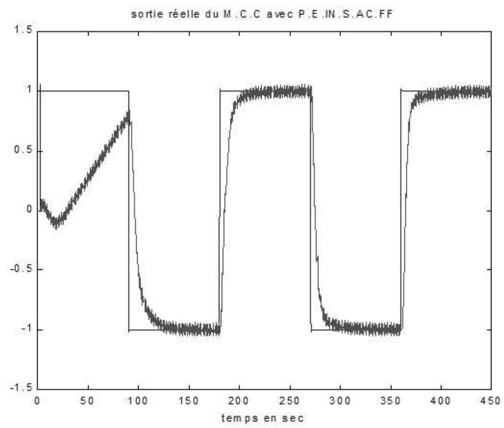
(a)



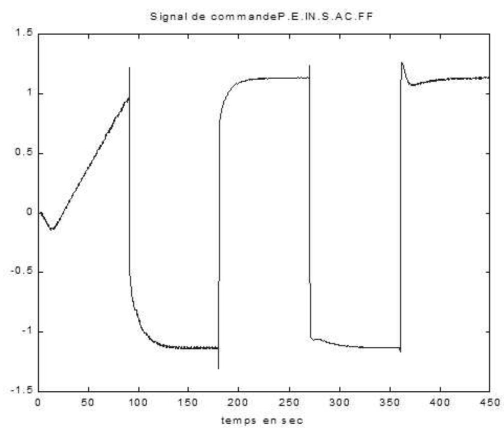
(b)

Figure 3: Process output with integer feedforward

6. Simulation Example



(a)



(b)

Figure 4: Process output with fractional feedforward

6. Simulation Example

Remarks

- The command signal u is more polish in the fractional case witch is a very useful property in regulation problem.

- The proposed fractional order configuration of feedforward maintains stability and at less the same level of performances, witch confirms the interest of integrating fractional strategy in robust adaptive control.

7. Conclusion

- In this paper we have presented a new robust adaptive control strategy, by introducing simple fractional feedforward configuration in the MRAC algorithm.
- The concept of positive realness condition which is the basis of this robust control strategy is extended to fractional order control systems.
- The stability proofs of this adaptive control scheme developed for integer order filters in control literature still holds for such systems.
- Simulation results have shown a better filtering ability of command and output signals, and more robustness against additive perturbations, than in the integer order feedforward configuration case.

Thank you