Commande - Robotique
Ordres Non Entiers

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CRONE OBSERVER
Definition and design methodology

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Outline

CRONE observer, definition and design methodology

1 – Dynamic output feedback based observer

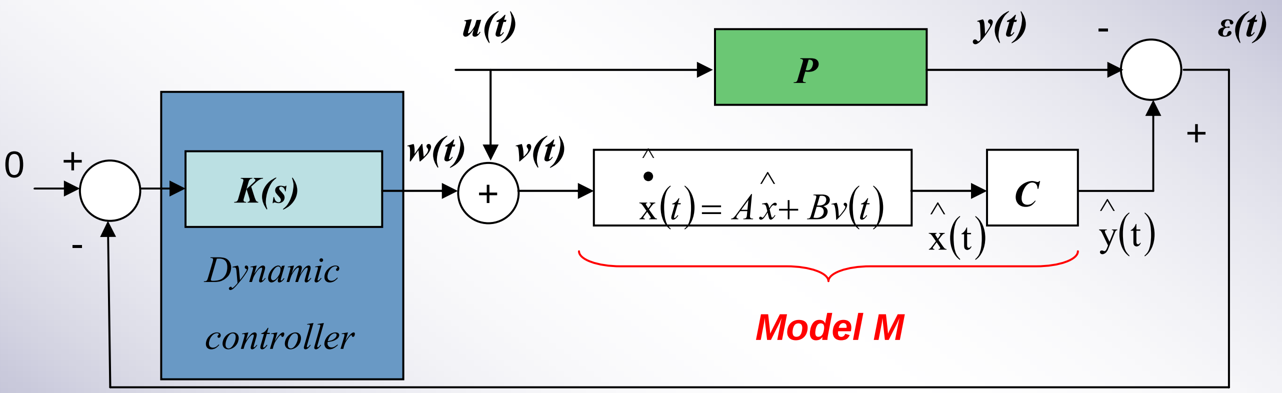
2 – CRONE Control System Design (CSD) principles

3 – CRONE observer

4 – Conclusion

Presentation

Dynamic output feedback based observer concept was introduced in (Marquez, 2003). The observation problem was solved using the feedback diagram



$$K:\begin{cases}\dot{x}_K(t)=A_K x_K(t)-B_K\varepsilon(t)\\ \qquad\quad =A_K x_K(t)-B_K C\left(x(t)-\hat{x}(t)\right)\\ w(t)=C_K x_K(t)\end{cases}\qquad P:\begin{cases}\dot{x}(t)=Ax(t)+Bu(t)\\ y(t)=Cx(t)\end{cases}\qquad M:\begin{cases}\dot{\hat{x}}=A\hat{x}(t)+Bv(t)\\ \quad =A\hat{x}(t)+B\left(u(t)+w(t)\right)\\ \hat{y}(t)=C\hat{x}(t)\end{cases}$$

$x(t)$ not measurable

$\hat{x}$ the estimated state

Presentation

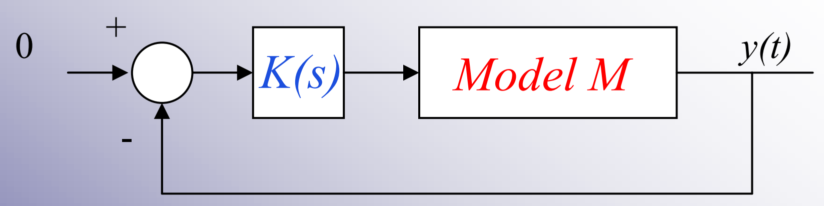
Dynamics observation error, is given by

$$\begin{aligned}\dot{\chi}(t) &= \dot{x}(t) - \dot{\hat{x}} \\ &= Ax(t) + Bu(t) - A\hat{x}(t) - B(u(t) + w(t)) \\ &= A\chi(t) - BC_K x_K(t)\end{aligned}$$

whose state space description is

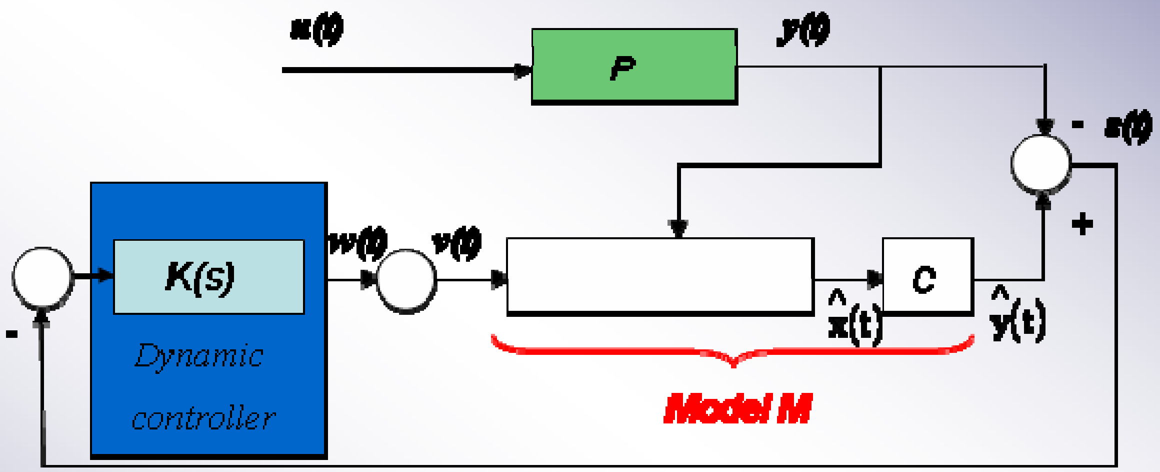
$$\begin{bmatrix}\dot{\chi}(t) \\ \dot{x}_K(t)\end{bmatrix} = \begin{bmatrix} A & -BC_K \\ B_K C & A_K\end{bmatrix}\begin{bmatrix}\chi(t) \\ x_K(t)\end{bmatrix} = A_o\begin{bmatrix}\chi(t) \\ x_K(t)\end{bmatrix}$$

A_0 is also the state matrix of the following closed loop system.



Internal stability => observation error cancellation

Extension to state observation with unknown input



P and M are described by the following state space description:

$$P:\begin{cases}\dot{x}(t)=Ax(t)+Bu(t)\\ y(t)=Cx(t)\end{cases}$$

$$M:\begin{cases}\dot{z}=Nz(t)+Bv(t)\\ \hat{x}(t)=z(t)-Ey(t)\\ \hat{y}(t)=C\hat{x}(t)\end{cases}$$

Extension to state observation with unknown input

Observation error dynamics is thus defined by:

$$\dot{\chi}(t) = Ax(t) - NECx(t) + ECAx(t) - N\hat{x}(t) + Bu(t) + ECBu(t) - Bv(t).$$

Suppose now that the matrix E is such that

$$B + ECB = 0 \quad or \quad E = -B(CB)^{*}, \qquad \text{if } (CB)^{*} \text{ exist.}$$

*: pseudo inverse

then

$$\dot{\chi}(t) = Ax(t) - NECx(t) + ECAx(t) + N\hat{x}(t) - Bv(t).$$

Using the state space description of the controller K

$$\begin{cases} \dot{\chi}(t) = Ax(t) - NECx(t) + ECAx(t) + N\hat{x}(t) - BC_K x_K(t). \\ \dot{x}_K(t) = A_K x_K(t) + B_K C\chi(t) \end{cases}$$

Extension to state observation with unknown input

Let now

$$P = I + EC \text{ and thus } EC = P - I$$

then

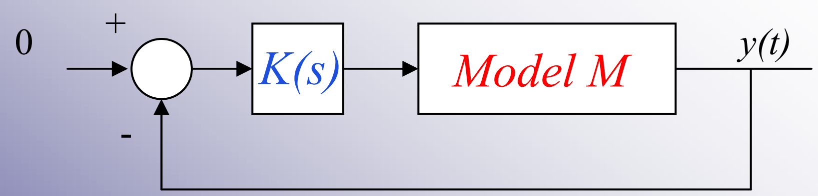
$$\begin{aligned} A - NEC + ECA &= A - N(P - I) + (P - I)A \\ &= -NP + N + PA \end{aligned}$$

If it is imposed now that

$$-NP + PA = 0 \text{ and thus } N = PAP^{-1},$$

it can be written

$$\begin{bmatrix} \dot{\chi}(t) \\ \dot{x}_K(t) \end{bmatrix} = \begin{bmatrix} N & -BC_K \\ B_K C & A_K \end{bmatrix} \begin{bmatrix} \chi(t) \\ x_K(t) \end{bmatrix}.$$



Internal stability => observation error cancellation

Introduction to fractional integro-differentiation

Let *y(t)* be the order *n* derivative of the causal signal *x(t)*

$$y(t) = x^{\{n\}}(t) = D^n x(t), D : \text{differentiation operator}$$

The transfer function of the linear operator D^n is defined by the Laplace transform:

$$D(s) = \frac{L\{y(t)\}}{L\{x(t)\}} = s^n.$$

Its impulse response is given by:

$$d(t) = L^{-1}\{s^n\} = \frac{t^{-n-1}}{\Gamma(-n)} H(t).$$

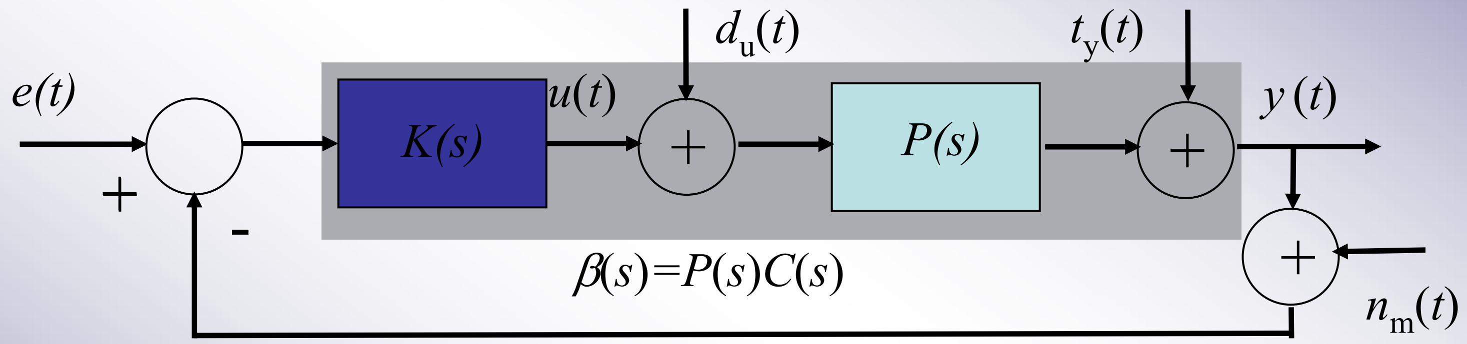
Convoluting *d(t)* and *x(t)*, *y(t)* can be computed using the following integrals:

$$y(t) = \int_0^t \frac{\theta^{-n-1}}{\Gamma(-n)} x(t-\theta) d\theta$$

If $Re[n] \in R^-$ and $Re[n] \neq 0$ which is the Riemann-Liouville definition, and

$$y(t) = \left(\frac{d}{dt}\right)^m \int_0^t \frac{\theta^{m-n-1}}{\Gamma(-n)} x(t-\theta) d\theta$$

Introduction to the CRONE methodology



Common CRONE control-system diagram

CRONE control methodology is based on the common unity feedback configuration.

Third generation CRONE Control

In the third generation CRONE control, the open loop behavior for the nominal parametric state of the plant is described by a transmittance based on complex fractional differentiation.

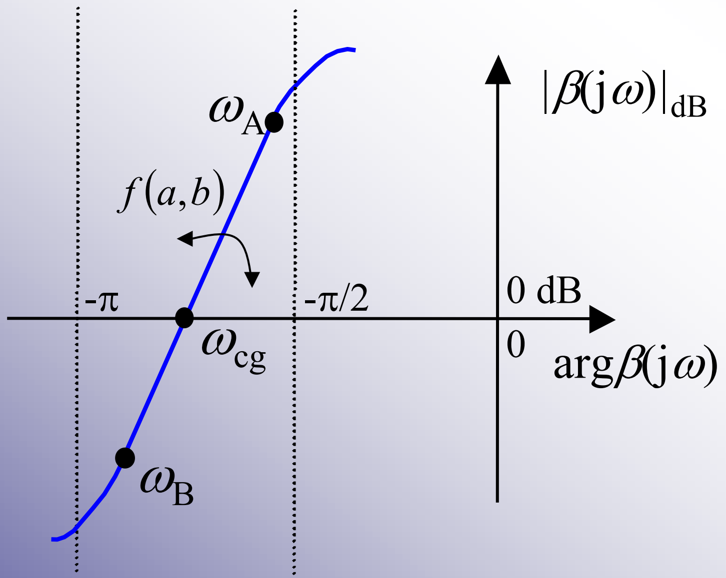
$$\beta(s) = \underbrace{K\left(\frac{\omega_l}{s}+1\right)^{n_l}}_{\text{Pseudo integrator}} \underbrace{\left(\frac{1+\frac{s}{\omega_h}}{1+\frac{s}{\omega_l}}\right)^{a}\left(\Re_{/i}\left[\left(C_0\frac{1+\frac{s}{\omega_h}}{1+\frac{s}{\omega_l}}\right)^{ib}\right]\right)^{-\text{sign}(b)}}_{\text{Complex band limited fractional integrator}} \underbrace{\frac{1}{\left(1+\frac{s}{\omega_h}\right)^{n_h}}}_{\text{Low pass filter}}$$

with $C_0 = \left(1+\left(\frac{\omega_r}{\omega_b}\right)^2 \Big/ 1+\left(\frac{\omega_r}{\omega_h}\right)^2\right)^{1/2}$

and with ω_l, ω_h and $K \in R^+$, n_l and $n_h \in N^+$.

Third generation CRONE Control

In the frequency range $[\omega_A, \omega_B]$ with $\omega_A > \omega_l$ and $\omega_B < \omega_h$, this open loop transmittance define a generalised template.



$$\beta(j\omega) = \left[\cosh\left(b\frac{\pi}{2}\right)\right]^{\mathrm{sign}(b)} \left(\frac{\omega_{cg}}{j\omega}\right)^{a} \left[\cos\left(b\ln\left(\frac{j\omega}{\omega_{cg}}\right)\right)\right]^{-\mathrm{sign}(b)}$$

$$\arg\beta(j\omega_{cg}) = -a\frac{\pi}{2}$$

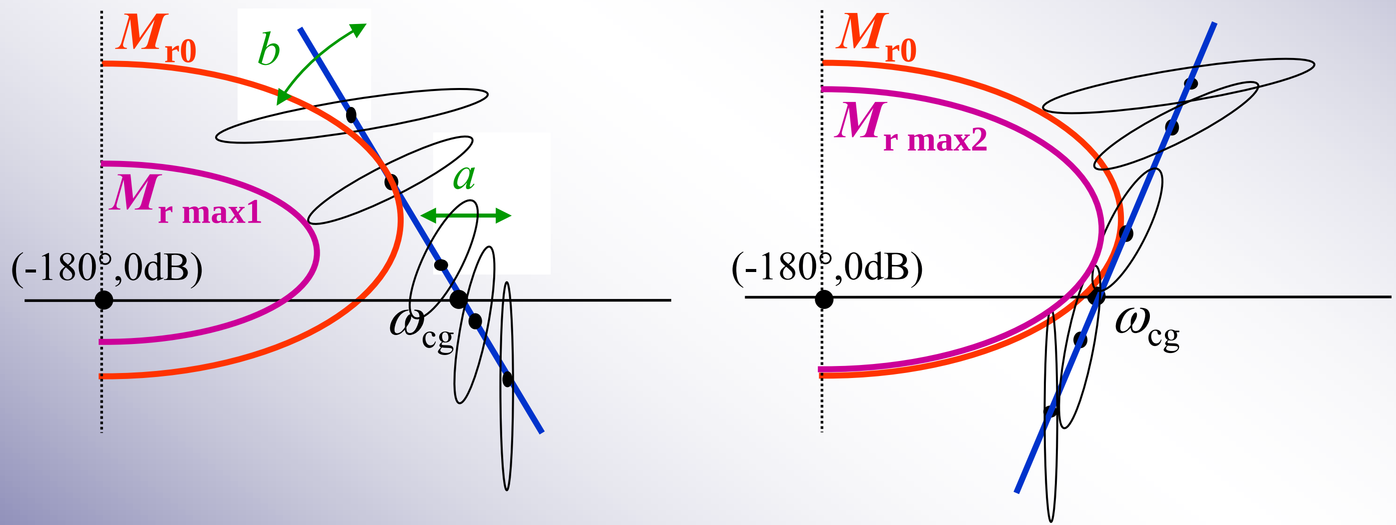
$$\left.\frac{d}{d\log_{10}\omega}|\beta(j\omega)|_{dB}\right|_{\omega=\omega_{cg}} = -20a$$

$$\left.\frac{d}{d\log_{10}\omega}\arg\beta(j\omega)\right|_{\omega=\omega_{cg}} = \mathrm{sign}(b)\ln(10)\tanh(b\,\pi/2)$$

In the Nichols chart at frequency ω_{cg}, the real order a determines the phase placement of the template, $a\pi/2$, and then the imaginary order b determines its angle to the vertical.

Third generation CRONE Control

The optimal template positions the open-loop uncertainty domains correctly, so that they overlap the low stability margin areas as little as possible.



Minimization of M_r *variations*

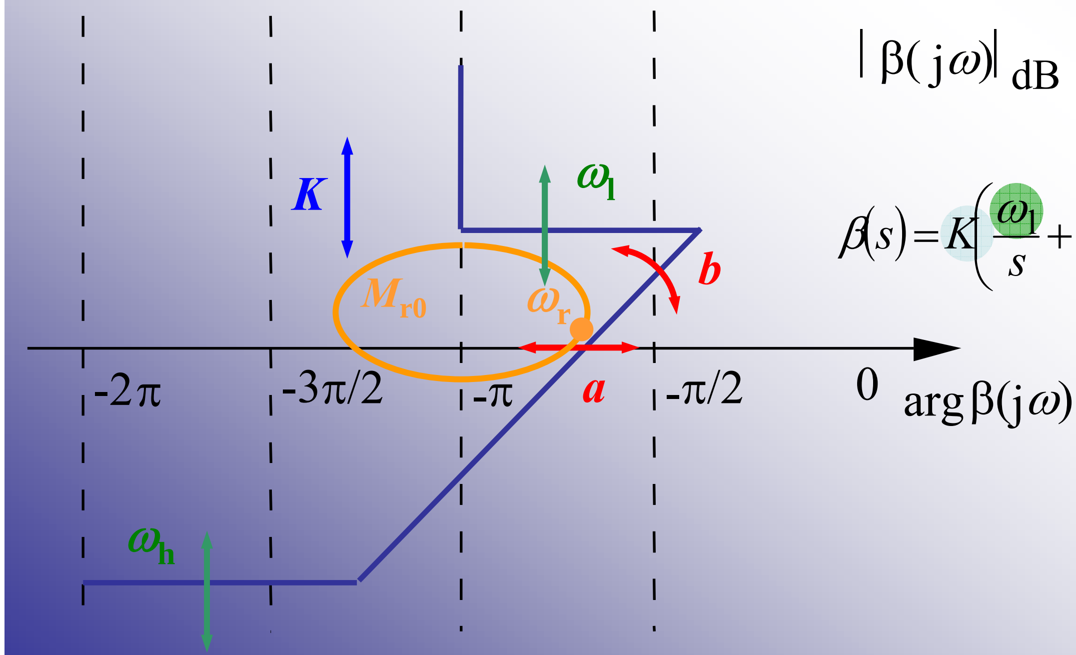
Generalized template ➲ Optimal template

$J_1 = M_{r\,max1} - M_{r0} \gg J_2 = Mr_{max2} - M_{r0}$

Third generation CRONE Control

n_l, n_h, a, b , ω_l, ω_h and K are 7 high-level parameters of the open-loop transfer function $\beta(s)$.

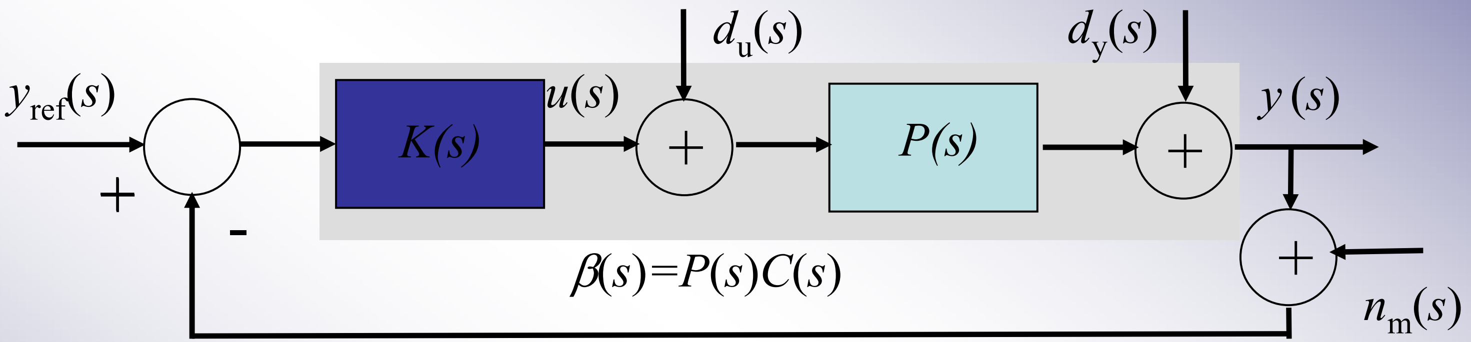
As n_l and n_h are fixed by the control-system designer, and as a tangency condition is required (thus 2 parameters will be dependent), only 3 independent parameters have to be optimized.



$$\beta(s) = K\left(\frac{\omega_l}{s}+1\right)^{n_l}\left(\frac{1+\frac{s}{\omega_h}}{1+\frac{s}{\omega_l}}\right)^{a}\left(\Re_{/i}\left[\left(C_0\,\frac{1+\frac{s}{\omega_h}}{1+\frac{s}{\omega_l}}\right)^{ib}\right]\right)^{-\mathrm{sign}(b)}\frac{1}{\left(1+\frac{s}{\omega_h}\right)^{n_h}}$$

Each parameter acts only on one shape feature of $\beta(j\omega)$, and thus can be optimized easily.

Third generation CRONE Control



To manage precisely performance related to tracking, regulation and control effort level, 5 inequality constraints are to be fulfilled for all plants (or parametric states of the plant) and for $\omega \in \mathbb{R}^+$:

$$\inf_P |T(j\omega)| \geq T_l(\omega) \quad \text{and} \quad \sup_P |T(j\omega)| \leq T_u(\omega)$$

$$\sup_P |S(j\omega)| \leq S_u(\omega)$$

$$\sup_P |KS(j\omega)| \leq KS_u(\omega)$$

$$\sup_P |PS(j\omega)| \leq PS_u(\omega)$$

$$T(s) = \frac{y(s)}{y_{ref}(s)} = -\frac{y(s)}{n_m(s)} = -\frac{u(s)}{d_u(s)} = \frac{K(s)P(s)}{1+K(s)P(s)}$$

$$S(s) = \frac{y(s)}{d_y(s)} = \frac{1}{1+K(s)P(s)}$$

$$PS(s) = \frac{y(s)}{d_u(s)} = \frac{P(s)}{1+K(s)P(s)}$$

$$KS(s) = \frac{u(s)}{y_{ref}(s)} = -\frac{u(s)}{n_m(s)} = -\frac{u(s)}{d_y(s)} = \frac{K(s)}{1+K(s)P(s)}$$

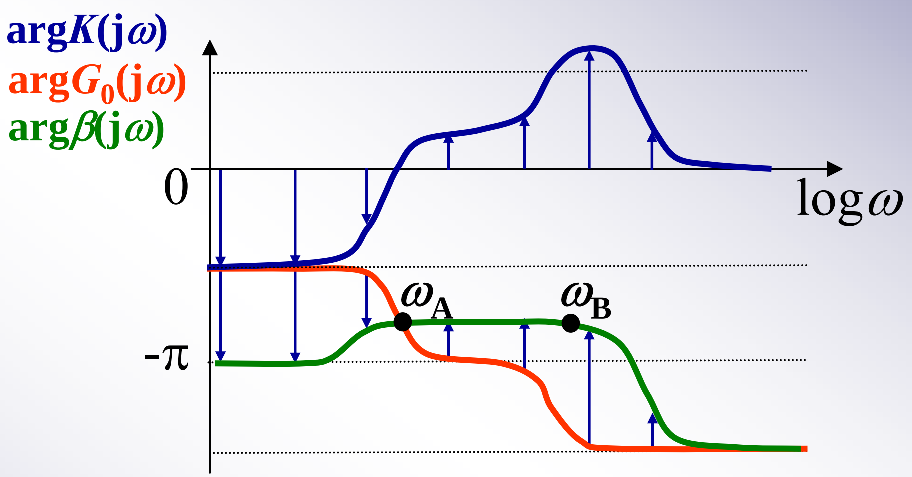
Third generation CRONE Control

A way to synthesize the rational form of the controller is to use its ideal frequency response

$$K_F(j\omega) = \beta(j\omega)/G_0(j\omega)$$

Identification of $K_F(j\omega)$ by a low-order transfer function $K_R(s)$

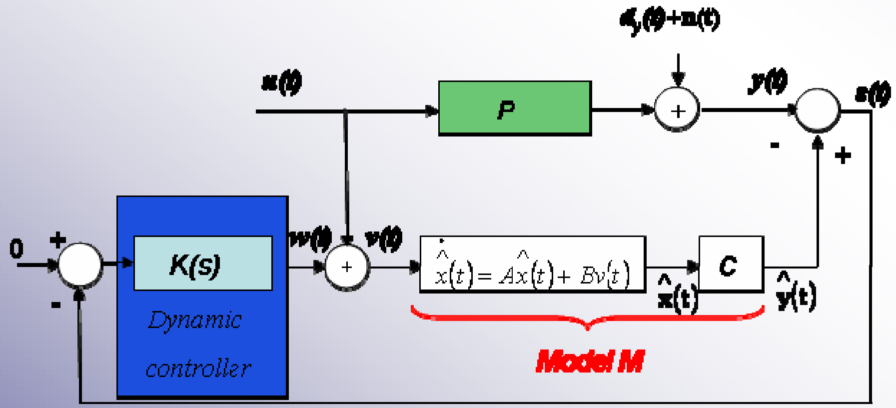
$$K_R(s) = \frac{C_0}{s^{N_{int}}} \frac{\prod_{i=1}^{n_1} 1 + \frac{s}{\omega_{z_i}} \prod_{i=1}^{n_2} 1 + \frac{2\zeta_{z_i} s}{\omega_{nz_i}} + \frac{s^2}{\omega_{nz_i}^2}}{\prod_{i=1}^{d_1} 1 + \frac{s}{\omega_{p_i}} \prod_{i=1}^{d_2} 1 + \frac{2\zeta_{p_i} s}{\omega_{np_i}} + \frac{s^2}{\omega_{np_i}^2}}$$



(use of a frequency-domain system-identification method)

Plant perturbations and disturbance effects

The plant, whose state is estimated, is submitted to perturbations. Effects of these perturbations but also effects of output disturbances $d_y(t)$ and measurement noises $n(t)$ on the estimation error are now studied.



$$M:\begin{cases}\dot{\hat{x}} = A\hat{x}(t)+Bv(t)=A\hat{x}(t)+B(u(t)+w(t))\\ \hat{y}(t)=C\hat{x}(t)\end{cases}$$

$$P:\begin{cases}\dot{x}(t)=(A+\Delta_A)x(t)+(B+\Delta_B)(u(t)+d_u(t))\\ y(t)=Cx(t)+d_y(t)+n(t)\end{cases}$$

Δ_A and Δ_B are real matrices of appropriate dimensions that models plant perturbations.

$$K:\begin{cases}\dot{x}_K(t)=A_Kx_K(t)-B_K\varepsilon(t)\\ \qquad =A_Kx(t)+B_K(Cx(t)+d_y(t)+n(t))\\ \qquad\qquad -B_KC\hat{x}(t)\\ w(t)=C_Kx_K(t)\end{cases}$$

Initial conditions

$$x_K(0)=0, x(0)=x_0, \hat{x}(0)=0$$

$$\Rightarrow \chi(0)=x_0=\chi_0$$

Plant perturbations and disturbance effects

Laplace transformation to preceding relations leads to:

$$M:\begin{cases}[sI-A]\hat{x}(s)=B(u(s)+w(s))\\ \hat{y}(t)=C\hat{x}(s)\end{cases}\qquad P:\begin{cases}[sI-(A+\Delta_A)]x(s)=\chi_0+(B+\Delta_B)u(s)\\ y(s)=Cx(s)+d_y(s)+n(s)\end{cases}$$

$$K:\begin{cases}x_K(s)=[sI-A_K]^{-1}\begin{pmatrix}B_K(Cx(s)+d_y(s)+n(s))\\ -B_KC\hat{x}(s)\end{pmatrix}\\ w(s)=C_Kx_K(s)\end{cases}$$

Difference of state equations of representation of M and P and using the state space representation of K gives :

$$[sI-(A+\Delta_A)]x(s)-[sI-A]\hat{x}(s)=\chi_0+(B+\Delta_B)u(s)-B(u(s)+C_Kx_K(s))$$

Plant perturbations and disturbance effects

We have *K(s)* the transfer function of controller *K*.

$$K(s) = C_K [sI - A_K]^{-1} B_K$$

Then preceding relation becomes:

$$\begin{aligned}[sI - (A + \Delta_A)]x(s) - [sI - A]\hat{x}(s) = \chi_0 + (B + \Delta_B)u(s) - Bu(s) \\ - BC_K [sI - A_K]^{-1} B_K C(x(s) - \hat{x}(s)) \\ - BC_K [sI - A_K]^{-1} B_K (d_y(s) - n(s)),\end{aligned}$$

and thus

$$\begin{aligned}[sI - (A + \Delta_A)]x(s) - [sI - A]\hat{x}(s) = \chi_0 + (B + \Delta_B)u(s) - Bu(s) \\ - BK(s)C(x(s) - \hat{x}(s)) \\ - BK(s)(d_y(s) - n(s)).\end{aligned}$$

Plant perturbations and disturbance effects

Laplace transform of the observation error is thus given by:

$$\begin{aligned}\chi(s) = {} & [sI - A + BK(s)C]^{-1}\chi_0 \\ & + [sI - A + BK(s)C]^{-1}\Delta_A x(s) \\ & + [sI - A + BK(s)C]^{-1}\Delta_B u(s) \\ & - [sI - A + BK(s)C]^{-1} BK(s)\left(d_y(s) + n(s)\right)\end{aligned}$$

CRONE observer synthesis

Without disturbances and plant perturbations (Δ_A=0, Δ_B=0, $d_y(s)+n(s)$=0)=> observation error converges towards 0 if the eigen values of $[sI-A+BK(s)C]^{-1}$ lie in the left half complex plane.

Moreover controller $K(s)$ can be used to minimizes the modulus of the elements transfer matrix $[sI-A+BK(s)C]^{-1}\Delta_A$ and vectors $[sI-A+BK(s)C]^{-1}\Delta_B$ and

$[sI-A+BK(s)C]^{-1}BK(s)$ in order to minimize observation error.

CRONE observer synthesis

An algorithm for the CRONE observer synthesis can thus be summarized as fallows:

- Choice of an open-loop gain-crossover frequency ω_{cg} that ensures a satisfactory observation error dynamics;

- Choice of order n_l and n_h in order to ensure that the gain of the elements of matrix

$[j\omega I\text{-}A+BK(j\omega)C]^{-1}\Delta_A$ and vectors $[j\omega I\text{-}A+BK(j\omega)C]^{-1}\Delta_B$ and $[j\omega I\text{-}A+BK(j\omega)C]^{-1}BK(j\omega)$ tend towards 0 as ω tends towards 0 and infinity to ensure a cancellation of observation error in steady stage and an immunity of this error to measurement noise;

- Optimization of parameters of open loop transmittance through the minimization of the criterion

$$J = \left\| \left[F_1(j\omega) \quad F_2(j\omega) \quad F_3(j\omega) \right] \right\|$$

With

$$F_1(j\omega) = W_A(\omega)\left[j\omega I - A + BK(j\omega)C\right]^{-1}\Delta_A$$

$$F_2(j\omega) = W_B(\omega)\left[j\omega I - A + BK(j\omega)C\right]^{-1}\Delta_B$$

$$F_3(j\omega) = W_C(\omega)\left[j\omega I - A + BK(j\omega)C\right]^{-1}BK(j\omega)$$

- Synthesis of the controller $K(s)$ using the preceding procedure.

Example-System definition

Let a nonlinear system modeled by the two differential equations

$$\begin{cases} \dot{i} = \frac{1}{L}\left(u(t) - Ri(t)\right) \\ \ddot{y} = \alpha \frac{i^2(t)}{y(t)} - 10 - \dot{y}(t) \end{cases}$$

with

$$1 \leq y(t) \leq 10$$

R, L, α are uncertain parameters

Around an operating point $y(t)=y_0$, its nominal ($R=R_0=1$, $L=L_0=0.1$ *and* $\alpha=\alpha_0=1$) and linearized model is

$$\begin{cases} \dot{x}_1(t) = -\frac{R_0}{L_0} x_1(t) + \frac{1}{L_0} u(t) \\ \dot{x}_2(t) = x_3(t) \\ \dot{x}_3(t) = 2\alpha_0 \frac{x_{10}}{x_{20}} x_1(t) - \alpha_0 \frac{x_{10}^2}{x_{20}^2} x_2(t) - x_3(t) \\ y(t) = x_2(t) \end{cases}$$

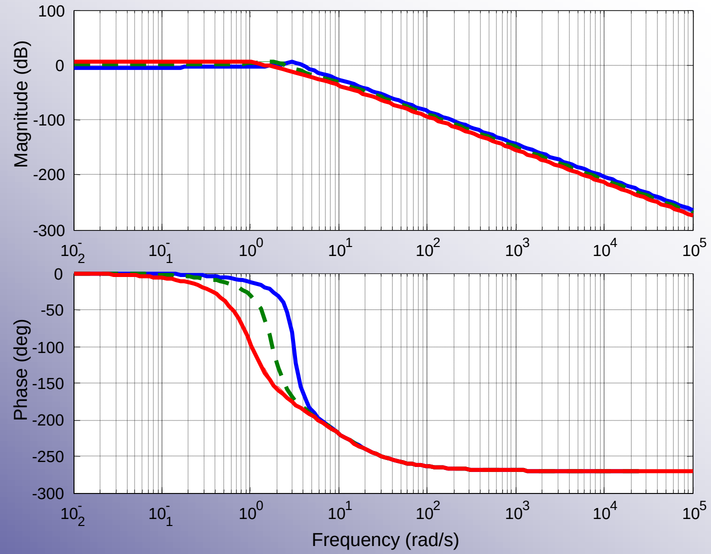
where

$$\begin{cases} x_1(t) = i(t), x_2(t) = y(t), x_3(t) = \dot{y}(t) \\ x_{10} = i_0 = \left(\frac{10}{\alpha_0} y_0\right)^{1/2} \\ x_{20} = y_0 \end{cases}$$

x_1(t): state variable to be estimated

Example-System definition

The figure shows the Bode diagrams of the perturbed linear plan *G(s)* defined by *N*=11 transfer functions $G_i(s)$ determined for values of y_0 between 1 and 10. From the 11 $G_i(s)$, the « nominal » plant $G_0(s)$ is that determined for $y_0 = \sqrt{10}$



(__) lower and greatest magnitude and phase of $G_i(j\omega)$.

(_ _) magnitude and phase of $G_0(j\omega)$

Example-performance specification

The level of the high-frequency measurement noise

$$n_m \approx \pm 10^{-3}$$

The allowable level of the noise on the estimated state

$$x_{1e} \approx \pm 10^{-1}$$

Then, the observation-loop bandwidth has to be limited and the controller $K(s)$ has to be such that

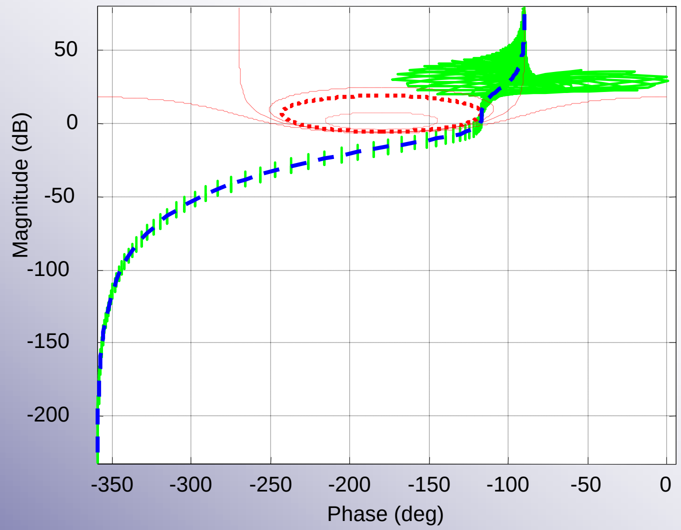
$$\lim_{\omega > 10^3} \left| \frac{K(j\omega)}{(R_0 + j\omega L_0)(1 + K(j\omega)G_i(j\omega))} \right| <\approx 10^2, \text{for } 1 \le i \le N \text{ and } G_i(s) \in G$$

If ω is such that $|K(j\omega)G_i(j\omega)| << 1$, as $R_0/L_0 = 10 << 10^3$, contraint becomes

$$\lim_{\omega > 10^3} |K(j\omega)| <\approx 10\omega$$

Example-open loop synthesis

A third generation CRONE controller is designed to control the perturbed plant *G*. The 3 independent and optimal parameters lead to the open-loop definition:



(__) frequency uncertainty-domains

(- - -) nominal open loop nichol locus

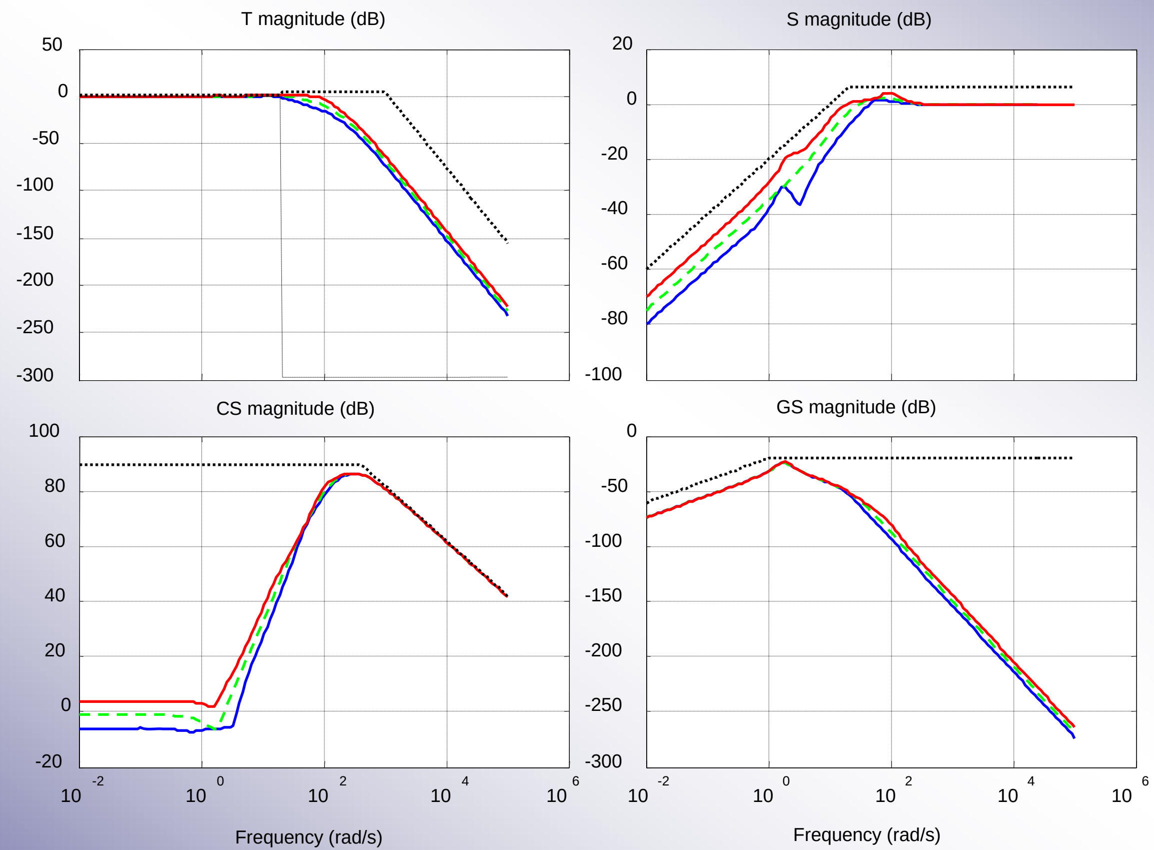
(…) 1dB M-contour

$$K = 6.90 \quad \omega_0 = 7\,{rad}/{s} \quad C_0 = 1.18$$
$$b_0 = 0.46 \quad \omega_l = 160\,{rad}/{s} \quad \omega_r = 15\,{rad}/{s}$$

By minimizing the cost function, the optimal template positions the uncertainty domains so that the overlap the 1dB M-contour as little as possible

Example- sensitivity functions

The sensitivity functions meet the constraints:



Nominal(---) and extrem (___) closed-loop sensitivity function

sensitivity function constraints(…)

Example- controller synthesis

When the optimal and the nominal open-loop transfer is determined, the fractional controller $K_F(s)$ is syntheses.

The rational integer model on which the parametric estimation is based, is given by

$$K_R(s) = \frac{0.01347e^{-4}s^4 + 0.9066s^3 + 15.44s^2 + 18.89s + 48.27}{1.11e^{-9}s^5 + 8.383e^{-7}s^4 + 0.0002259s^3 + 0.02544s^2 + s}.$$

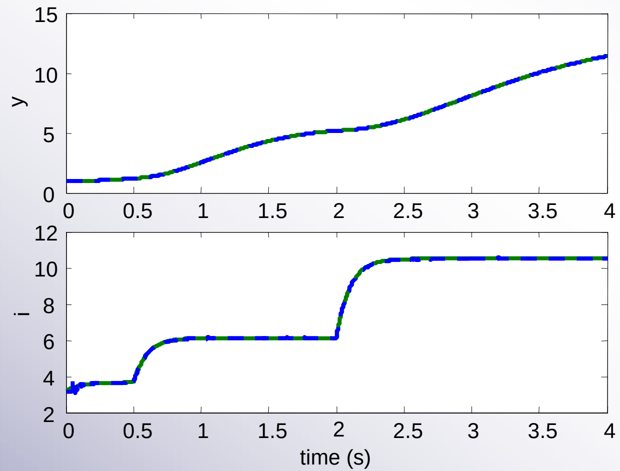
Time-domain simulation are achieved for four configurations of nonlinear system:

1. $R = R_0, L = L_0$ and $\alpha = \alpha_0$
2. $R = R_0 + 30\%, L = L_0 - 20\%$ and $\alpha = \alpha_0$
3. $R = R_0 - 30\%, L = L_0 + 20\%$ and $\alpha = \alpha_0$
4. $R = R_0, L = L_0$ and $\alpha = \alpha_0 - 20\%$

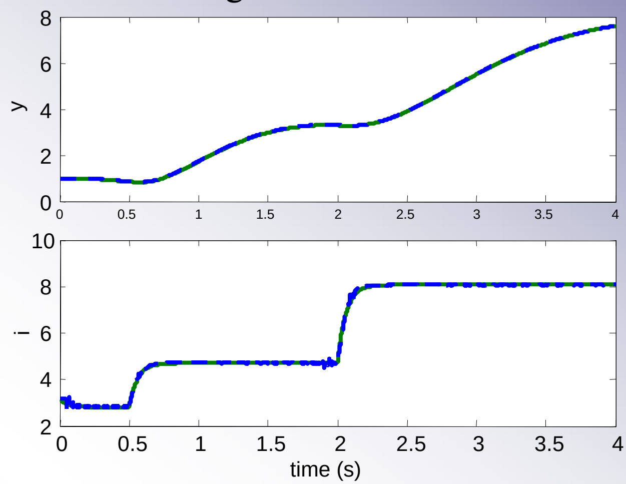
Input signal $u(t)$ is defined such as, for the nominal and non-disturbed system, output $y(t)$ would go successively from 1 to $10^{0.5}$ and then to 10. Input disturbance $d_u(t)=0.5$ and the measurement noise $n_m(t)$ is provided through a quantizer block applied on y with a quantization interval of 10^{-3}

Example- time simulation

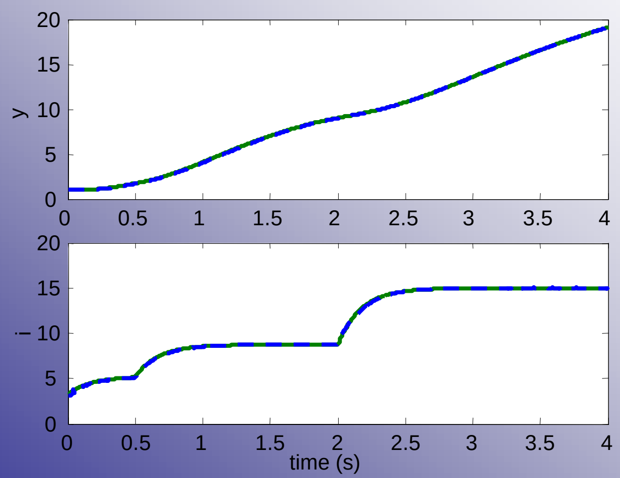
Configuration #1



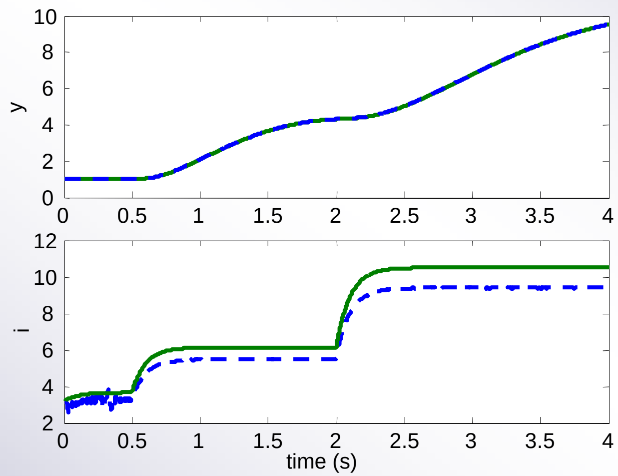
Configuration #2



Configuration #3



Configuration #4



Time response of the output $y(t)$ and the state $i(t)$ of nonlinear system (____) and of the observer (---)

Example-Results

Conclusion:

The time domain simulations #1 to #3 shows:

- good steady state and dynamic performance of the observer.
- a good work of the observer even if output y goes outside the range [1, 10]

Time domain simulation #4 shows:

- even if the controller cancel the output error, there is a bias between i(t) and i_e(t)

↳ improvement ⟶ This bias could be cancelled.

- by using an estimates or measured value of parameter α within the nonlinear model
- in the performance/robustness trade-off framework, a LPV or multimodel approach could also be used to design a controller

Development of a dynamic output feedback based observer, referred to as a CRONE observer:

This name results in the introduction of CRONE controller in a feedback loop whose goal is to cancel the error between a model state and the unmeasured state error of plant that must be estimated. Such an approach of state observation permits:

- a generalization of the Luenberger form that allows more freedom and flexibility in the design
- a formulation allowing more transparent view of the observer properties in term of feedback elements
- to pose the disturbance rejection problem and the observation robustness problem in the context of robust control theory.

The main difference between this paper and (Marquez, Riaz 2005) are:

- the extension of the dynamic output feedback based observer idea to the observation problem with unknown input
- the use of a CRONE controller to solve disturbance rejection problem and the observation robustness